

MODEL ORDER REDUCTION FOR LARGE SCALE FINITE ELEMENT ENGINEERING MODELS

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Abstract. *Software MOR for ANSYS has been developed at IMTEK in 2003. It allows us to perform model reduction directly to finite element models developed in ANSYS. The goal of the present paper is to describe progress achieved for the last two years and review our publications with application of MOR for ANSYS to various engineering problems for different domains: heat transfer, structural mechanics, thermomechanical models, and acoustics including fluid-structure interaction. We also discuss computational scalability of model reduction and the advanced development such as parametric and weakly nonlinear model reduction.*

1 INTRODUCTION

Finite element models enjoy widespread use in engineering. However, the discretization in space leads to a high-dimensional system of ordinary differential equations and its transient and harmonic analysis takes too much time even with modern hardware.

The goal of model order reduction is to find a low-dimensional but accurate approximation of the large-scale dynamic system. This way one can drastically reduce time required for transient and harmonic simulation and find a compact representation suitable for system level simulation.

During the last ten years, considerable progress has been achieved in model order reduction of a linear system of ordinary differential equations [1]. From a computational viewpoint, the most advantageous is implicit moment matching [2] and we have chosen this method for software MOR for ANSYS that has been developed in 2003. Our goal was to allow engineers to use new model reduction methods developed by mathematicians directly for finite element models developed at typical industrial environment. As such, we have targeted ANSYS, as it is quite popular among engineers.

In 2004, the design of MOR for ANSYS has been described during PARA'04 conference [3] and then a review of first results for thermal and structural models has been presented at

MNTS 2006 [4]. The goal of the present paper is to describe progress achieved for the last two years after Ref [3][4] have been written. In Sections 2 to 8, we review the application of MOR for ANSYS to different physical domains. In Section 9 we discuss the computational scalability of implicit moment matching via the Arnoldi process. Finally in Sections 10 and 11, we present the advanced development related to parametric and weakly nonlinear model reduction.

2 HEAT TRANSFER

A thermal problem is the easiest one for model reduction as the discretization of the heat transfer partial differential equation leads to a system of ordinary differential equations of the first order. In this case, one can use moment matching methods directly, as originally they have been developed for first order systems. The majority of examples for the heat transfer are related to electro-thermal MEMS [5] but, in our view, the conclusions are applicable to other thermal applications as well.

In thesis [6], model reduction has been used to a variety of microhotplate-like devices. The main conclusion was that model reduction is a very efficient way to find an accurate low-dimensional compact thermal model that approximates well dynamic behavior of the original high-dimensional problem. The results show that a thermal model with about hundred thousands nodes can be accurately described by a reduced system with about 30 generalized coordinates.

Model reduction via implicit moment matching does not have global error estimates. As a result, the choice for an optimal dimension for the reduced system is an open question. In [7], an error indicator for the Arnoldi process has been developed. The mathematical proof is still missing, but in our experience the error indicator effectively solves the problem of choosing the dimension of a reduced system in engineering environment.

The problem of coupling reduced model between each other has been discussed in [8]. It happens that this seemingly simple problem has not been solved so far and requires more research.

3 TREATMENT OF PROPORTIONAL DAMPING

The discretization of partial differential equations for physical problems described below leads to a system of ordinary differential equations of the second order. The transformation of such a system to the first order is always possible but undesirable. First, this breaks the internal structure of the system; second, such a transformation increases the dimension of the state vector twice. Special algorithms, so called second order Arnoldi, have been recently developed to employ moment matching directly for a second order system [9][10].

Proportional damping is the special case when the damping matrix is a linear combination of the mass and stiffness matrices, $E = \alpha M + \beta K$. It is very popular in engineering because in this case eigenvectors remain the same as for the undamped system when the damping matrix is set to zero. The coefficients α and β are considered to be of empirical nature and one of simulation requirements is to perform simulation for different numerical sets of α and β .

In [11][12], a model reduction scheme has been suggested when model reduction is first

made for the undamped system (like in [13]) but after that the damping matrix is projected onto the same low-dimensional subspace. Along this way, the coefficients α and β are preserved in the symbolic form and can be changed at the level of the reduced model.

Recently, a mathematical proof in [14] has explained empirical results in [11][12]. It has been proved that in the case of proportional damping the subspace for the undamped system matches moments independently of α and β . Because of such a lucky coincidence one can preserve α and β in the symbolic form for free.

4 STRUCTURAL MECHANICS

Model reduction is well developed for linear systems and hence applications in structural mechanics are limited by small deformation analysis. As a result, vibrational simulation is one of natural applications for model reduction. Model reduction has been used to make vibrational analysis of a knuckle in [15]. Model reduction based on implicit moment matching has been also compared with that based on the mode superposition.

Three-dimensional serial reconstruction techniques allow us to construct very detailed micro-finite element (micro-FE) model of bones that can represent the porous bone micro-architecture [16]. Solving these models, however, implies solving very large sets of equations (order of millions degrees of freedom), which inhibits harmonic response analyses. It was shown for bone models up to 1 million degrees of freedom that formal model order reduction allows us to perform harmonic response simulation for such problems very efficiently [17]. The results suggest a new multiscale strategy: accurate 3D modeling with computed tomography followed by model reduction.

5 SMALL SIGNAL ANALYSIS FOR RF-MEMS

Modeling of a microelectromechanical system [18] requires coupling of structural mechanics with electrostatics. The simplest approach is to describe the electrostatic force by means of lumped capacitors (TRANS126 element in ANSYS). Yet, even in this case the model is already nonlinear.

Small signal analysis, that is, linearization of a nonlinear model around an operation point, is an evident option to apply linear model reduction. Common practice in the area of RF-resonators is to use a bias voltage and then a small harmonic signal to operate the device. Pre-stressed simulation in ANSYS is designed to model this behavior. During nonlinear static analysis, the effect of the bias voltage is included into the stress-stiffening matrix and the latter is used during harmonic response simulation. The methodology to incorporate model reduction into this process for RF-resonators is presented in [19][20].

6 THERMOMECHANICAL MODELS

Thermal stresses influence behavior of a mechanical structure. Provided that in addition to small deformations one assumes the temperate independent secant coefficient of thermal expansion, linear model reduction should work for a coupled thermomechanical problem. Model reduction for a thermomechanical ANSYS model with inhomogeneous temperature

distribution has been performed in [21]. An additional problem is that ANSYS makes coupling between structural and mechanical subsystems through the load vector and the coupling matrix is not available directly. Yet, it was possible to evaluate the coupling matrix from element load vectors by assuming that the element thermal strain is caused by average element temperature.

In our view, possible applications of model reduction for thermomechanical models can be related to the development of ultraprecision machines [22]. Design of such machines is impossible without real-time error compensation for many side effects, among which deformation due to inevitable thermal gradients plays the central role. Modern trend to develop appropriate control systems includes simulation of finite element thermomechanical models. Model reduction can reduce the dimension of finite element models and develop compact accurate thermomechanical models for system-level simulation automatically.

7 ACOUSTICS WITH FLUID-STRUCTURE INTERACTIONS

Computational acoustics is mostly limited to a linear problem and, as such, well suited for model reduction. For example, in a modern passenger vehicle or a commercial airplane, the noise, vibration and harshness (NVH) performance is one of the key parameters, which the customer uses to assess product quality. To this end, acoustics simulation is indispensable to evaluate the low frequency NVH behavior of automotive/aircraft interiors during the design phase.

ANSYS uses Cragg's pressure formulation that leads to unsymmetric system matrices. This increases computational requirements but otherwise does not prevent the use of model reduction. In [23], two case studies have been chosen: a clamped undamped aluminum plate backed by a rigid walled rectangular cavity and a model structure, made of simple beams and plates. The results showed that implicit moment matching implemented in MOR for ANSYS is an efficient means to generate accurate compact structural-acoustic models.

8 MODEL REDUCTION AS FAST SOLVER

As was mentioned at the beginning, moment matching via Krylov subspaces is very efficient computationally. The computational analysis performed in [3] led to the conclusion that model reduction time for MOR for ANSYS is comparable with that of a static solution provided that a direct solver can be used and there is enough memory to keep the factor. This means that model reduction can be considered as a fast solver for transient and harmonic response simulation.

Computational results in [6][15][19][21][23] confirm this conclusion. They show that it is much faster first to perform model reduction with MOR for ANSYS and transient/harmonic simulation with a reduced model rather than to make full-scale transient/harmonic simulation in ANSYS.

Based on this fact, model reduction has been used as a fast solver within an optimization loop in [12] to make an optimal design for a microaccelerometer. In [24], a similar procedure has been employed in order to determine unknown materials properties from experimental measurements.

9 COMPUTATIONAL SCALABILITY OF MOR FOR ANSYS

Computational experiments have shown that modern multifrontal direct solvers are quite competitive for 3D finite element models provided that there is enough memory to keep the factor. In our experience, for computers with 4 Gb of RAM the upper level of the problem dimension is about 500 000 degrees of freedom [3]. However, 4 Gb was insufficient to keep in memory matrices and the factor for a bone model with about one million degrees of freedom [17]. In this case, out-of-core solver was required and this slowed down the model reduction process.

In order to keep efficiency of model reduction for larger models a parallelizable sparse direct solver needs to be implemented on a parallel system. Recently, the multifrontal massively parallel sparse direct solver MUMPS has been benchmarked [26] on SGI Origin 3800 in respect to four micro finite element bone models in the range from 1 to 12 million degrees of freedom [25]. The results showed that MUMPS is a good candidate to include it in the MOR for ANSYS to make it scalable for parallel systems.

10 PARAMETRIC MODEL REDUCTION

It is desirable to preserve some parameters in system matrices in the symbolic form during the model reduction process and this happens to be possible by means of multivariate moment matching. Our work in this respect was recently reviewed in [27] and here we cite the two journal papers only. In [28], the film coefficient has been preserved in the symbolic form for a thermal model. In [29], parametric model reduction has been used to speed up transient simulation of voltammograms for a microelectrode.

11 MODEL REDUCTION FOR WEAKLY NONLINEAR MODELS

Automatic model reduction for a nonlinear system in general is still at frontiers of science. In a special case, when nonlinearity comes from temperature dependent material properties, a finite element model can be classified as weakly nonlinear. This means that the dependence on the state vector is mostly quadratic or cubic. In this case, there is a special approach that generalizes moment matching to treat additional quadratic and cubic terms [30]. Recently, it has been successfully applied to a thermal model with nonlinear film coefficient [31]. One of the problems here is to extract nonlinear system matrices from finite element software. In [31], this was possible due to a very simple structure of the problem but, unfortunately, the question how to do it in the general case remains unanswered.

12 CONCLUSIONS

A low-dimensional subspace that captures accurately dynamics of the original system does exist for many finite element models and implicit moment matching allows us to find it very efficiently for different physical domains.

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