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ERROR INDICATOR TO AUTOMATICALLY GENERATE DYNAMIC COMPACT PARAMETRIC THERMAL MODELS

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ABSTRACT

During recent years, several groups have shown that parametric model reduction is possible in general. In particular, it has been shown that it allows us to generate compact thermal models while preserving film coefficients as parameters. Unfortunately, in order to run algorithms, a user should specify how many moments and what type to generate in advance and there were no formal rules to this end but a "trial and error" approach. We present an approach based on a local error in the transfer function and show that it can automate the process to build a dynamic compact parametric thermal model in much greater extent. We demonstrate that our algorithm can preserve three film coefficients for the first thermal model and material properties such as heat conductivity and heat capacity for the second model.

1. INTRODUCTION

Model reduction is a rapidly developing area of mathematics [1]. It allows us to take a high-dimensional finite element model developed at device level simulation and convert it efficiently into a low-dimensional approximation for system level simulation [2]. Model reduction approaches have been successfully applied to a thermal problem to automatically generate a dynamics compact thermal model [3][4][5][6][22].

However, in its original form model reduction does not allow us to preserve parameters in the system matrices that naturally arise in many applications. Fortunately, a new development, that is, parametric model reduction, allows us to overcome this limit.

In our knowledge, the first work on parametric model reduction has been presented by Weile et al [7] in 1999 and applied to describe frequency depended surfaces in [8]. This approach has been generalized from two to many

parameters in [9] and in parallel re-discovered in [10][11][12]. We have suggested an empirical solution to a similar problem in [13] and an alternative algorithm in [14]. Note that different authors use different names for the same method: multiparameter model reduction in [9], multidimensional model reduction in [10][11] and multivariate model reduction in [12]. Our choice in this respect is parametric model reduction as it allows us to preserve parameters in system matrices in the symbolic form.

In [12][14][15], this approach has been successfully applied to a thermal problem when film coefficients have been preserved as symbols in a reduced model. Although these works have demonstrated that this is the right way to go, an important practical question remains unanswered. That is, how to choose moments to include into the reduced model. A straightforward approach to choose some order and then generate all the moments up to this order does not scale well with the number of parameters [9]. For example, if we choose to preserve four film coefficients then a reduced model made from all first derivatives has the dimension of 6, a reduced model made from all second derivatives has the dimension of 21, and a reduced model from all third derivatives already has the dimension of 56 (see Appendix F in [9]). At the same time, we may need derivatives of higher order than three to describe accurately transient behavior of the original model.

The explosion in the dimension of a reduced model is due to mixed moments (mixed derivatives). In [15] the authors have observed that one can actually ignore mixed moments in the case of a thermal problem and proved this by numerical simulation. However, even in this case it is unclear how to choose the number of moments along each parameter automatically. Although time in the form of the Laplace variable formally looks like the film coefficient in the transfer function, we may need more moments along the time axis. In [15] the authors have limited themselves

to a stationary problem and have not researched this problem further.

The use of local error estimators has been researched in [16][17] (see also discussion in [2]). Error indicators for Arnoldi-based model reduction have been suggested in [18]. In the present paper, we use these results as inspiration for a heuristic procedure suited for parametric model reduction. We suggest an approach that controls the dimension of the reduced model automatically based on local error control. We apply the approach to two thermal models and report our numerical observations. First is a thermal model of a microthruster unit [19] (see also [13][14]) where the goal is to preserve three film coefficients. Second is a thermopile based IR detector [21] when a compact thermal model should preserve material properties of the gas in the symbolic form.

2. OVERVIEW OF PARAMETRIC MODEL REDUCTION

Let us briefly review the application of parametric model reduction to a thermal problem. The discretization in space by the finite element/volume/difference method of the heat transfer equation leads to a system of ordinary differential equations as follows

$$\left(E + \sum_i q_i E_i\right) \frac{dT(t)}{dt} + \left(K + \sum_i p_i K_i\right) T(t) = Bu(t), \quad (1)$$

$$y(t) = CT(t)$$

where $T(t)$ is the vector of unknown temperatures at the nodes. E and K are the heat capacity and heat conductivity system matrices, B is the input matrix, and C is the output matrix. The vector u comprises input functions such as heat sources. The output matrix specifies particular linear combinations of temperatures that of interest to an engineer. Our goal is to preserve the parameters q_i and p_i in the symbolic form in the reduced model (a film coefficient or material property). A parameter contributes to the global system matrix by means of the matrix E_i or K_i .

The transfer function of (1) can be expressed as follows

$$H(s) = C \left\{ s(E + \sum_i q_i E_i) + K + \sum_i p_i K_i \right\}^{-1} B, \quad (2)$$

and in addition to the Laplace variable s it contains the parameters q_i and p_i .

Model reduction is based on an assumption that there exists a low-dimensional subspace V that accurately enough captures the dynamics of the state vector $T(t)$:

$$T \approx Vz. \quad (3)$$

In order to find such a subspace V that does not depend on parameters to preserve, the transfer function (2) can be treated as a function in many variables (s , q_i and p_i) and one can perform its multivariate expansion. Then V is taken as a subspace that spans multivariate moments of (2) (see [9]-[15]). This way, V does not depend on parameters in (1) and (2).

Provided V is known, one obtain a low-dimensional model by projecting (2) on V as follows

$$\begin{aligned} \{V^T E V + \sum_i q_i V^T E_i V\} \frac{dz(t)}{dt} + \\ \{V^T K V + \sum_i p_i V^T K_i V\} z(t) = V^T B u(t) \end{aligned} \quad (4)$$

Eq (4) preserves the original parameters in the symbolic form and as a result we refer to this approach as parametric model reduction.

3. LOCAL ERROR CONTROL

We have limited ourselves to the Single-Input-Single-Output case when the transfer function (2) is a scalar, the input matrix B is a vector and the output matrix C is a single row. Another simplification is that we ignore mixed moments following the observation in [15]. As a result, the model reduction algorithm is practically equivalent to that described in [15] except that we take into consideration the Laplace variable as well.

We assume that a user specifies the range of interest for the frequency and parameters:

$$\begin{aligned} s_{\min} < s < s_{\max} \\ q_{i,\min} < q_i < q_{i,\max} \\ p_{i,\min} < p_i < p_{i,\max} \end{aligned} \quad (5)$$

Additionally a user chooses expansion points for the Laplace variable and parameters. In our work, we take the expansion point for the Laplace variable as zero because this allows us to preserve the stationary state. Finally, a user also specifies the error ε for the maximum frequency of interest f_{\max} ($s_{\max} = i2\pi f_{\max}$). In our experience, 1% error for 100 Hz corresponds to reasonably accurate approximation in the time domain although this may depend on the frequency spectrum of the input function.

Our main idea is to choose the number of vectors in the Krylov subspaces in such a way that the difference between the transfer functions of the original and reduced models at the maximum frequency is below of the specified level for the allowable range of parameters as follows

$$\|H(s_{\max}, q_i, p_i) - H_{\text{reduced}}(s_{\max}, q_i, p_i)\| < \varepsilon, \quad (6)$$

We present numerical results in the next two sections.

4. CASE STUDY I: PRESERVING FILM COEFFICIENTS

We have used the thermal problem from [13][14]. It is described in [19] and available on-line in the Matrix Market format [20] at <http://www.imtek.uni-freiburg.de/simulation/benchmark>.

Fig. 1 shows the device. The heat is generated by a heater and propagates through the device to three boundaries (top, side, bottom). Each boundary has its own film coefficient and the goal is to preserve them in the symbolic form. As a result, Eq (1) is written as

$$E \frac{dT(t)}{dt} + (K + h_t K_t + h_s K_s + h_b K_b) T(t) = Bu(t), \quad (7)$$

$$y(t) = CT(t)$$

where h_t , h_s and h_b are the film coefficients at the top, side and bottom respectively (they replace p_i in Eq 1). The dimension of the full model is 4257.

We have chosen the same expansion point for all film coefficients as $h_{t,0} = 10$ and the same maximum value of 10^6 . The maximum frequency was chosen as 100 Hz and required accuracy at this frequency is 1%.

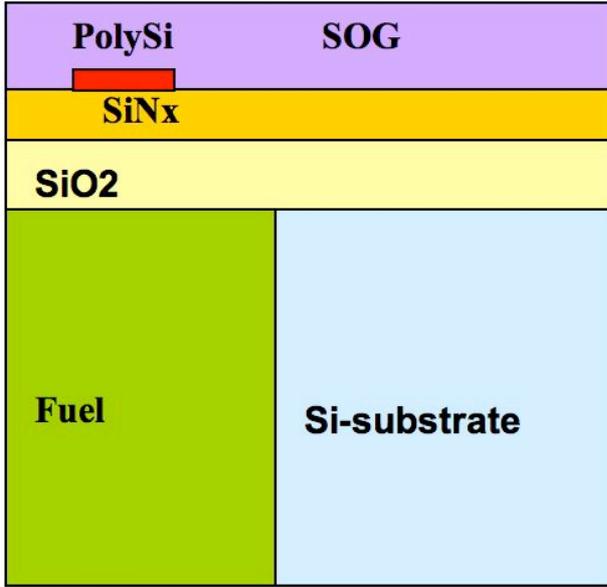


Fig. 1. A 2D-axisymmetrical model of the micro-thruster unit (not scaled). The axis of the symmetry on the left side. A heater is shown by a red spot.

As the mixed moments have been neglected, it was necessary to generate four subspaces along the Laplace variable and the three film coefficients. The first subspace has been made along the Laplace variable. We have used the value of the transfer function at $H[s_{\max} = 100, h_{t,0} = 10, h_{s,0} = 10, h_{b,0} = 10]$ to keep the local error below 1%. It was necessary to generate 28 vectors to reach desired accuracy. The next subspace has been generated along h_t and the value of the transfer function at $H[s_{\max} = 100, h_t = 10^6, h_{s,0} = 10, h_{b,0} = 10]$ has been used for the local error control. It took 13 vectors to reach the convergence. At this point, it happened that the two next subspaces along h_s and h_b were unnecessary as the convergence in respect to $H[s_{\max} = 100, h_t = 10^6, h_s = 10^6, h_{b,0} = 10]$ and $H[s_{\max} = 100, h_t = 10^6, h_s = 10^6, h_b = 10^6]$ has been reached simultaneously while generating vectors for the

second subspace. As result, the dimension of the reduced model was 41. The convergence history is shown in Fig. 2.

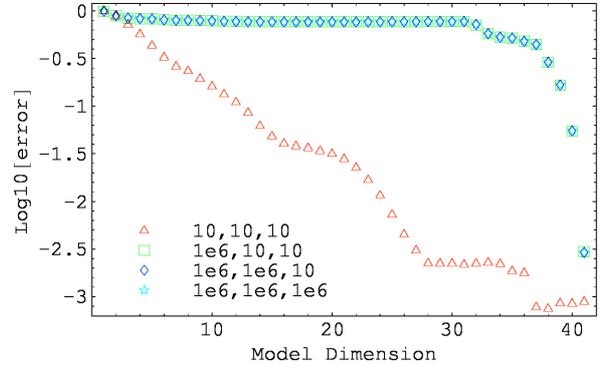


Fig. 2. Convergence history when the order to generate subspaces was s , h_t , h_s and h_b . Different markers show the error at different values of the transfer function $H[s_{\max} = 100, h_t, h_s, h_b]$, where the values of the film coefficients shown near to the marker.

It is worthy of noting that when we have changed the order to generate subspaces to s , h_b , h_s and h_t , the final result was the same although the convergence history became different (see Fig. 3). In this case, it happened that generated along the Laplace variable reached the convergence limit not only in respect to $H[s_{\max} = 100, h_{t,0} = 10, h_{s,0} = 10, h_{b,0} = 10]$ but also for $H[s_{\max} = 100, h_{t,0} = 10, h_{s,0} = 10, h_b = 10^6]$ and $H[s_{\max} = 100, h_{t,0} = 10, h_s = 10^6, h_b = 10^6]$. Then we needed a subspace along h_t only and the final dimension of the reduced model was again 41.

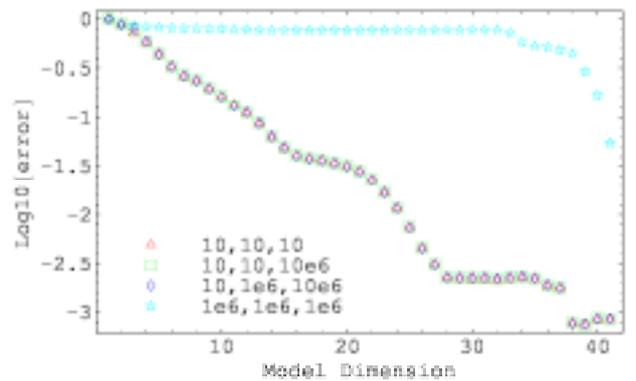


Fig. 3. Convergence history when the order to generate subspaces was s , h_b , h_s and h_t . Different markers show the error at different values of the transfer function $H[s_{\max} = 100, h_t, h_s, h_b]$, where the values of the film coefficients shown near to the marker.

Table 1. Simulation results for the reduced model as compared with the original model.

h_{top}	h_{side}	h_{bottom}	Stationary solution	Error (%) in stationary	Error (%) in transient
1	1	1	832.	0.11	0.47
1	1	100	85.8	0.0033	0.21
1	1	10000	39.2	0.21	0.24
1	100	1	50.0	0.044	0.15
1	100	100	47.8	0.031	0.15
1	100	10000	39.1	0.19	0.22
1	10000	1	38.3	0.066	0.17
1	10000	100	38.3	0.066	0.17
1	10000	10000	38.3	0.068	0.17
100	1	1	73.4	0.0031	0.20
100	1	100	55.4	0.00089	0.17
100	1	10000	36.8	0.17	0.21
100	100	1	43.6	0.027	0.15
100	100	100	42.4	0.020	0.15
100	100	10000	36.7	0.16	0.21
100	10000	1	36.0	0.056	0.17
100	10000	100	36.0	0.056	0.17
100	10000	10000	36.0	0.058	0.17
10000	1	1	7.56	0.000008	0.18
10000	1	100	7.56	0.000011	0.18
10000	1	10000	7.56	0.00016	0.18
10000	100	1	7.56	0.000007	0.18
10000	100	100	7.56	0.000003	0.18
10000	100	10000	7.56	0.00016	0.18
10000	10000	1	7.56	0.00068	0.18
10000	10000	100	7.56	0.00068	0.18
10000	10000	10000	7.56	0.00067	0.18

Simulation results for the reduced model of dimension 41 and the original model of dimension 4257 are shown in Table 1. For 27 cases of different values of film coefficients, we present the stationary temperature at the heater, the error in the stationary state made by the

reduced model and the error in the transient step response as compared with the original model. The error for transient response was estimated as follows:

$$error = \left[\sum_{i=1}^n (T_i - \hat{T}_i)^2 \right]^{1/2} / \left[\sum_{i=1}^n T_i^2 \right] \quad (8)$$

where $T = (T_1, T_2, \dots, T_n)$ is the transient solution of the original system (7) and $\hat{T} = (\hat{T}_1, \hat{T}_2, \dots, \hat{T}_n)$ is the transient solution of the reduced model (4).

Fig. 4 to 6 show the transient response of the original model (red) and reduced model (green) for the three different cases from Table 1. The difference is very small and comparable to the line thickness.

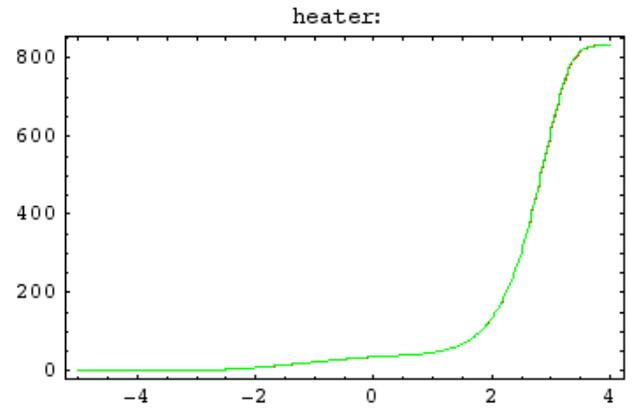


Fig. 4. The transient response for the step input function: temperature vs. $\log_{10}[\text{time}]$ for the case when $h_t = 1$, $h_s = 1$ and $h_b = 1$.

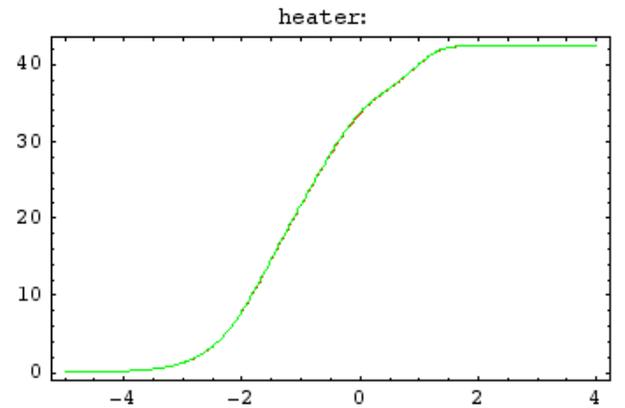


Fig. 5. The transient response for the step input function: temperature vs. $\log_{10}[\text{time}]$ for the case when $h_t = 100$, $h_s = 100$ and $h_b = 100$.

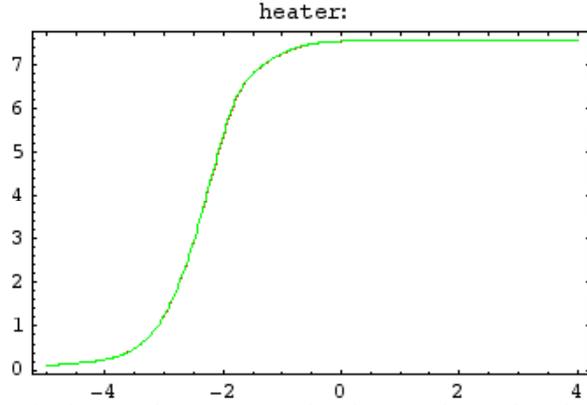


Fig. 6. The transient response for the step input function: temperature vs. $\log_{10}[\text{time}]$ for the case when $h_t = 10000$, $h_s = 10000$ and $h_b = 10000$.

5. CASE STUDY II: PRESERVING MATERIAL PROPERTIES

A thermopile based IR detector can be fabricated with CMOS compatible micromachining processes [21] (see Fig. 7). The sensor is formed by a silicon wafer with a thermally isolated membrane that contains the thermocouples hot junction. The cold junctions are placed in the opposite side of the membrane, over the silicon bulk, to assure a maximum thermal isolation with respect to the hot junctions. A silicon absorber is located in the center of the membrane with the thermocouples hot junctions. When the absorber is heated up, a temperature difference appears between the hot and cold junctions. Due to the Seebeck effect this temperature difference produces a voltage difference that is the output signal of the sensor. For calibration or test purposes, a heater is also placed above the absorber with the corresponding electrical contacts. Due to the thermal isolation the main thermal flux from the hot junctions is to the surrounding gas. Moreover, the output signal of the device is quite sensitive to the thermal properties of this gas. This opens new opportunities for its use to detect different gases.

The sensor operation can be modeled by a thermal model however the gas material properties change and they should be preserved in the symbolic form. In other words, Eq (1) becomes

$$(E + cE_c) \frac{dT(t)}{dt} + (K + \kappa K_\kappa) T(t) = Bu(t), \quad (9)$$

$$y(t) = CT(t)$$

where there are two gas-specific parameters: the heat conductivity κ and the heat capacity per unit volume c .

The thermal sensor model in the form of Eq (9) has been made in ANSYS and it had dimension of 2870. The goal of model reduction was to find a low-dimensional approximation and at the same time to preserve κ and c . We have chosen required range of the heat conductivity and heat capacity to cover air, nitrogen, neon, argon, krypton, and xenon.

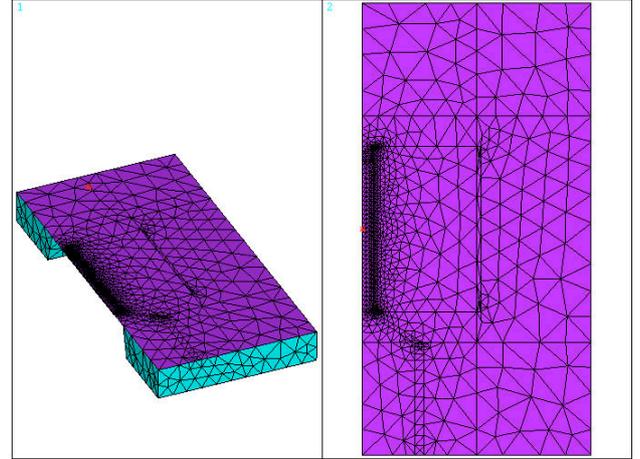


Fig. 7. Micromachined thermopile based IR detector.

Parametric model reduction in this case happened to be easier as the range of parameter values is much more narrow than in the previous case. We needed to generate 10 vectors along the Laplace variable and then only one vector along each parameter. Thus, the dimension of the reduced system was equal 12.

Simulation results for the reduced model of dimension 12 and the original model of dimension 2870 are shown in Table 2. For six gases, we present the stationary temperature at the hot and cold junction, the error in the stationary state made by the reduced model and the error in the transient step response as compared with the original model. Fig. 8 and 9 show the step response of the full scale (red) and reduced models (green) for the nitrogen: the difference is close to the line thickness.

Table 1. Simulation results for the reduced model as compared with the original model.

Gas	Junction	Stationary solution	Error (%) in stationary	Error (%) in transient
air	hot	0.00925	0.016	0.82
	cold	0.000014	0.52	1.23
nirtrogen	hot	0.00933	0.015	0.83
	cold	0.000014	0.49	1.19
neon	hot	0.00706	0.083	0.78
	cold	0.000012	2.56	3.69
argon	hot	0.0105	0.0038	0.84
	cold	0.000015	0.12	0.88
krypton	hot	0.0122	0.000021	0.86
	cold	0.000017	0.00068	0.74
xenon	hot	0.0131	0.0013	0.87
	cold	0.000017	0.040	0.79

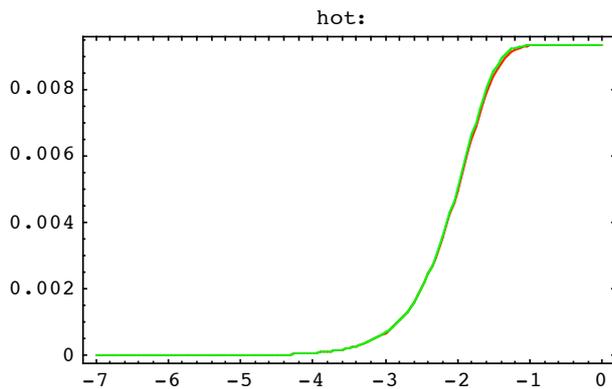


Fig. 8. The transient response for the step input function: temperature vs. $\log_{10}[\text{time}]$ for the hot junction in the case of nitrogen.

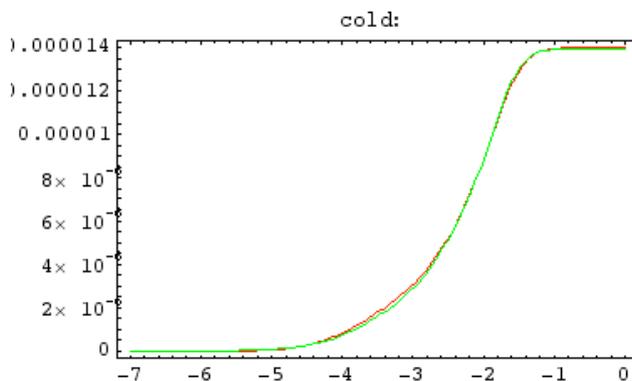


Fig. 9. The transient response for the step input function: temperature vs. $\log_{10}[\text{time}]$ for the cold junction in the case of nitrogen.

6. CONCLUSION

In our view, there are two main results of our study. 1) Mixed moments seem to be unnecessary indeed in agreement with [10][11][15]. 2) The local error control allows us to choose the right number of moments in order to give good approximation properties of the reduced model.

Unfortunately, at present it is hard to say how general these results are. No doubts, it is necessary to try more different thermal models to gain more experience.

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