Model Order Reduction of MEMS for Efficient Computer Aided Design and System Simulation

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Abstract

In the present paper we focus on the application aspect of Krylov-subspace-based model order reduction. We present the software mor4ansys that allows us to directly apply model reduction to ANSYS™ finite element models. We show that, for many MEMS thermal and structural mechanics problems, model reduction is very efficient means to generate compact models for system-level simulation. Finally, we discuss on how one can use model reduction during the engineering design process.

Introduction

Modern commercial computer aided design (CAD) packages, for example MEMSCAP and Coventor, attempt to support a range of physical models relevant for developers of micro electro- mechanical systems (MEMS) by adding finite and boundary element packages. As result, we have seen a proliferation of the use of models based on partial differential equations in designing individual devices. Yet there is a clear gap between device simulation based on physical models and system-level simulation that considers circuitry, control, and eventually packaging [1]. In principle, one can include device models, after their discretization in space, directly to the system-level simulation, yet the typical sizes of device models have caused a computational bottleneck, especially for "array" microsystems such as charge-coupled devices (CCDs) and digital micromirror devices (DMDs), where each device model with order $n \sim 10^5$ is required many times over. The alternative approach is to employ so called behavioural or compact device models, but the process to generate them is more art than science.

In the present paper, we discuss how model order reduction techniques [2][3][4] can be successfully used to efficiently create compact models of a variety of MEMS devices for use at the system-level simulation and design. We take ANSYS, a popular commercial finite element package, as the device simulator and describe results obtained by means the software ***mor4ansys*** (pronounced "more for ANSYS") that was recently developed by us at IMTEK.

We start with a short description of the software and then we review the results obtained. We stress specific engineering requirements to model reduction of second order systems and finish by addressing the use of the finite elements model during the design flow.

mor4ansys

The developed software (http://www.imtek.uni-freiburg.de/simulation/mor4ansys/) comprises two almost independent modules. The first package reads binary ANSYS files and assembles a dynamic system in the form of

$$\begin{aligned} E\dot{\boldsymbol{x}} &= A\boldsymbol{x} + B\boldsymbol{u} \\ \boldsymbol{y} &= C\boldsymbol{x} \end{aligned} \qquad (1)$$

for first order systems, or

$$M\ddot{\boldsymbol{x}} + E\dot{\boldsymbol{x}} + K\boldsymbol{x} = B\boldsymbol{u}$$
$$\boldsymbol{y} = C\boldsymbol{x} \tag{2}$$

for second order systems, where A, E, M, K are the system matrices, B is the input and C the output matrices.The second module applies the Block Arnoldi algorithm [5] to Eq (1) or (2) to find a low-dimensional subspace V

$$\boldsymbol{x} = V\boldsymbol{z} + \varepsilon \tag{3}$$

such that it allows us to reproduce the transient behaviour of the original state vector with required accuracy. After that, the original equations are projected to the subspace found, for example for Eq (1) we have $E_r = V^T E V$, $A_r = V^T A V$, $B_r = V^T B$, $C_r = CV$.

The software can also read as well as write the matrices for the original system in Matrix Market format [6]. More detail is given elsewhere [7].

Model reduction results for the thermal structural mechanics models

For a thermal problem, the application of model reduction is straightforward as the discretization produces Eq (1) directly. Engineering aspects of model reduction for thermal problems are discussed in Ref [8]. We have applied model reduction to several devices in order to develop so called dynamic compact thermal models:

- Microthruster unit [9],
- Thermal model for two chips positioned on a PCB-board [10],
- Hot-plate gas sensor [11],
- Microanemometer, a flow meter based on convective thermal flow [12],
- Chip cooled by air flow [12].

The first three models include heat dissipation by diffusion only, while the last two add a convective heat flow term. In all cases, the reduced model of order 30 was able to describe the original model of dimension up to 100 000 with an accuracy of a few percent. This is consistent with results obtained in this area by other groups [13][14][15], that is, that model reduction can be considered as a very efficient tool to generate small models for system-level simulation. Note that, although model reduction is limited to linear systems, the input function may depend on temperature and model reduction allows us to preserve this relationship [8][11].

ANSYS includes model reduction based on the Guyan method [16]. In Ref [17], the clear advantage of Krylov-based model reduction over Guyan, in the case of a thermal problem, has been demonstrated.

The second order system, Eq (2), can in principle be converted to a first order system. Yet there are many advantages to preserve its structure during model reduction [18][3]. There is increased interest among mathematicians: for example, three from seven talks in this session is devoted to this problem. However, in structural mechanics the damping matrix often is modelled as Rayleigh damping

$$E = \alpha M + \beta K \tag{4}$$

even the nature of the damping matrix is in reality much more complex (squeeze film damping, thermo elastic damping, etc.). Nevertheless, if view the parameters α and β in practice as fitting parameters, one can employ Eq (4) quite successfully. In this case, it is much more important to preserve these parameters during model reduction rather than to derive an optimal reduced model for particular values of α and β. Another consideration is based on engineering intuition that the damping matrix should not play the major role to find a good subspace V as the most essential information contains within the mass and stiffness matrices.

As result, in mor4ansys, the low-dimensional basis for Eq (2) is built as the orthogonolized Krylov subspace $\Im(K^{-1}M, K^{-1}\boldsymbol{b})$ based on the two matrices [19], while the damping matrix is ignored. The reduced dumping matrix has been computed as a projection in Eq (2), $E_r = V^T E V$, that in the case of Eq (4) is reduced to

$$E_r = \alpha M_r + \beta K_r \tag{5}$$

where subscript r denotes the "reduced" matrices. Note that the original Rayleigh parameters are preserved as symbols in the reduced models.

We have applied such a procedure to several MST devices:

- Boded wire [20],
- Microgyroscope [21],
- The radio frequency micro-switch [22],
- Microaccelerometer [23].

As a typical example, Fig. 1 to 4 displays the transient simulation and frequency response of the original and reduced models of the microgyroscope [21].

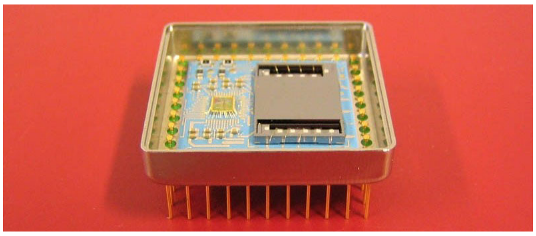

Figure 1: The Butterfly microgyroscope and μSIC mounted together. Courtesy of Dr. Dag Bilger, IMEGO, Sweden.

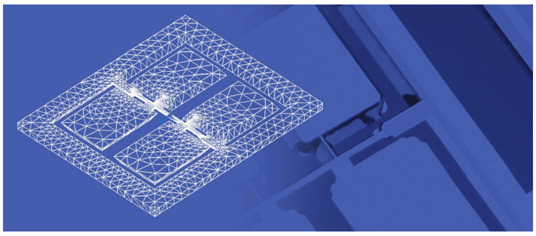

Figure 2: Finite element mesh of the gyro with a background photograph of the gyro wafer. Courtesy of IMEGO, Sweden.

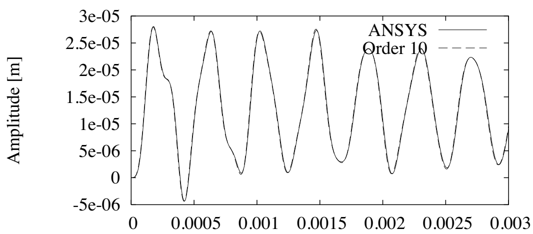

Figure 3: Comparison of transient behaviour for the full and reduced models.

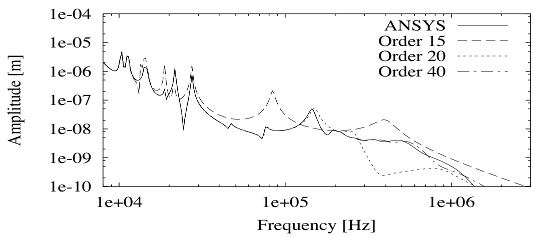

Figure 4: Comparison of transfer functions of the full and reduced models.

Similar to thermal models, a reduced model of dimension 30 allows us to represent the transient and harmonic response of the original model quite accurately and the Krylov subspace model reduction outperforms the Guyan method [20].

Model reduction for design

The main conclusion of our research is that model reduction works very well for thermal and structural mechanics finite element models. A reduced model of dimension 30 can accurately describe transient and harmonic response of high dimensional models up to dimension 300 000. Nonetheless, this is based on numerical experiments and does not exclude a possibility that one may need a reduced model of higher dimension. Thus, when model reduction is applied in practice special care should be taken to make sure that the chosen dimension of the reduced model is good enough.

On the other hand, the dimension of 30 is excessive in many of our cases. From an engineering viewpoint, in many cases a reduced model of dimension 10 or even 5 is already applicable. The main practical question nowadays becomes, how to choose the optimal dimension of the reduced model, as unfortunately the moment matching methods do not yet have a global error estimate that can be employed to make such a decision completely automatically [2]-[5].

In Ref [24], a local error estimate has been developed for the Lanczos algorithm. In spite of its local character, it may be good enough for practical application. However, quite often practitioners use the Arnoldi process as it is more numerically robust and allows us to preserve stability and passivity of the original model without extra computational cost. In this case, we have found empirically [25] that the difference between two neighboring reduced models allows us to estimate the total error and this finding seems to be a good solution for practical applications.

Provided that the question on how to choose the dimension of the reduced model is solved, one can use model reduction in the design process as shown in Fig. 5.

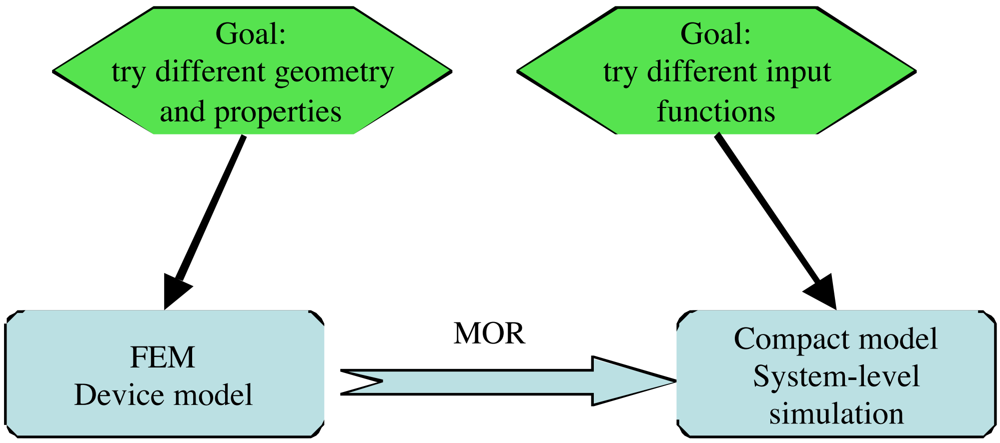

Fig. 5. Use of model reduction during design.

Model reduction can produce a compact model for system-level simulation and the latter can be used without changes for different input functions. In this case, one obtain enormous saving in the computational time. However, quite often an engineer would like to change the geometry or materials properties of the device in order to optimize its performance. In this case, the reduced model should be generated again after the change is incorporated in the original finite element model. In this case, the advantage of model reduction is clearly depends on the computational cost to generate the reduced model.

The computational cost of model reduction by means of mor4ansys is discussed in Ref [7]. Provided that the dimension 30 is enough for the reduced model and a direct solver can be employed, the cost of model reduction is comparable with that of a stationary solution for the original high-dimensional model. In our experience, the computational time for model reduction is at least ten times less than the time for a typical transient or harmonic simulation. This means that, even in the extreme case when the reduced model is used just once, model reduction can still save computational time considerably. For example, model reduction has been already used during geometry optimization of the micro-accelerometer [23] quite successfully.

Finally we would like to mention a new development that may allow us to enrich the reduced model by preserving design parameters during model reduction. We have already shown that in the case of the second order system one can preserve the Rayleigh parameters in the reduced model (see Eq (4) and (5)). In Refs [26][27], moment matching has been generalized to a multidimensional Padé-type approximation that, in addition to frequency, can include design parameters. A more primitive approach to the same problem has been presented in Ref [28]. Hopefully soon one will be able to solve both design goals shown in Fig. 5 with the same reduced model.

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