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Krylov Subspace Techniques for Low Frequency Structural Acoustic Analysis and Optimization

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ABSTRACT

A reduced order model is developed for low frequency, fully coupled, undamped and constantly damped structural acoustic analysis of interior cavities, backed by flexible structural systems. The reduced order model is obtained by applying a *Galerkin* projection of the coupled system matrices, from a higher dimensional subspace to a lower dimensional subspace, whilst preserving some essential properties of the coupled system. The basis vectors for projection are computed efficiently using the Arnoldi algorithm, which generates an orthogonal basis for the Krylov subspace containing moments of the original system. The key idea of constructing a reduced order model via Krylov subspaces is to remove the uncontrollable, unobservable and weakly controllable, observable parts without affecting the transfer function of the coupled system. The reduced order modelling technique is applied to a frame-panel two-way coupled vibro-acoustic optimization problem, with stacking sequences of the composite structure as design variables. The optimization is performed via a hybrid search strategy combining outputs from Latin Hypercube Sampling (LHS) and Mesh Adaptive Direct Search (MADS) algorithm. It is shown that reduced order modelling technique results in a very significant reduction in simulation time, while maintaining the desired accuracy of the optimization variables under investigation.

1 INTRODUCTION

Improving the acoustic behaviour of vehicle and aerospace interiors is an ever increasing challenge for manufacturers. It is now common practice to evaluate the low frequency noise, vibration, harshness NVH behaviour of vehicle or aerospace interiors using fully coupled finite element-finite element (FE/FE) or finite element-boundary element (FE/BE) discretized models at the design phase of the product development process. Due to the coupling between the fluid and structural domains in the FE/FE formulation, the resulting mass and stiffness matrices are no longer symmetrical. Such a multi disciplinary approach requires the solution of these coupled fluid and structural equations, causing an inevitable increase of computational time and expense [1]. Since there exists two forms of solution (coupled and uncoupled), it is often left to the engineer to decide on the approach best suited to the problem under investigation. However, a 'one-way' coupled analysis ignores the fluid loading on the structure, which is often the cause of cavity boom at low frequencies.

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Therefore, a fully coupled analysis is preferred in many vehicle/aerospace applications, but the computational time required to solve the higher dimensional model restricts its subsequent use. This is often the case when the structure is spatially damped i.e. different parts of the structure carry different damping values. Furthermore, this presents a major problem where optimization is required, especially when a large number of design variables are to be optimized.

The two most popular approaches currently used to reduce the computational time of such coupled problems are the mode superposition and the component mode synthesis (CMS) method. The former method uses the dominant natural frequencies and mode shapes, extracted from a normal modal analysis, and the response is assumed to be a linear combination of the modes. However, the reduction thus obtained is often not substantial. Further, the CMS method relies on the user to select interface nodes to enforce coupling conditions, which is a possible source of additional error. Other approaches to decrease computational time include generation of Ritz vectors, truncated coupled FE/FE analysis, and the patented Acoustic Transfer Vector (ATV) method, to name a few. The reader is referred to [1], for a review of some other approaches to reduce computational time. More recently, however, model order reduction (MOR) via implicit moment matching, has received considerable attention among mathematicians and the circuit simulation / Micro Electro-Mechanical Systems (MEMS) community [2, 3, 4, 5]. The aim of MOR is to construct a reduced order model, from the original higher dimensional model, which is a good representation of the system input/output behavior at certain points in the time or frequency domain. The reduction is achieved by applying a projection from a higher order to a lower order space using a set of Krylov subspaces, generated by the Arnoldi algorithm. Additionally, the reduced model preserves certain essential properties such as maintaining the second order form and stability.

The paper focuses on the application of such Krylov based MOR techniques to undamped and structurally damped, fully coupled structural acoustic problems. The rest of the paper is laid out as follows: In *Section- 2*, the general framework for model order reduction for second order systems is introduced. In *Section- 3* the Arnoldi procedure adapted for model order reduction for the coupled damped structural acoustic problem is described. In *Section- 4* a numerical example from is solved using the direct approach in ANSYS FE code and the MOR via Arnoldi approach. In *Section- 5* MOR is incorporated via the Arnoldi process in the structural-acoustic optimization process to speed up simulation time, whilst maintaining the desired accuracy of the objective function under investigation. *Section- 6* summarizes the paper with a short discussion of the results.

2 MODEL ORDER REDUCTION FOR SECOND ORDER SYSTEMS

After discretization of a general dynamical model of mechanical system, one obtains a system of second order ordinary differential equations in matrix form as follows:

$$\begin{gathered}[M]\ddot{x}(t)+[C]\dot{x}(t)+[K]x(t)=F\,u(t)\\ y(t)=L^T x(t)\end{gathered} \tag{1}$$

where *(t)* is the time variable, *x(t)* is the vector of state variables, *u(t)* is the input force vector, and *y(t)* the output measurement vector. The matrices *M*, *C* and *K* are mass, damping and stiffness matrices, F and L are the input distribution matrix and output measurement matrix at certain points respectively. A harmonic simulation, assuming $\{F\}=F_0 e^{j\omega t}$ and ignoring damping in (1) yields:

$$[-\omega^2[M]+[K]]\{x\}=\{F\}$$
$$y(\omega)=L^T x(\omega) \tag{2}$$

where, $\{\omega\}$ denotes the circular frequency, and $\{x\},\{F\}$ denote complex vectors of state variables and inputs to the system respectively. The principle of model reduction is to find a lower dimensional subspace $V \in \Re^{Nxn}$, and,

$$x=Vz+\epsilon \quad \text{where,} \quad z \in \Re^n, n \ll N \tag{3}$$

such that the time dependent behaviour of the original higher dimensional state vector $\{x\}$ can be well approximated by the projection matrix *V* in relation to a considerably reduced vector $\{z\}$ of order *n* with the exception of a small error $\epsilon \in \Re^N$. Once the projection matrix *V* is found, the original equation (2) is projected onto it. The projection produces a reduced set of system equations, as follows:

$$[-\omega^2[M_r]+[K_r]]\{z\}=\{F_r\}$$
$$y_r(\omega)=L_r^T z(\omega) \tag{4}$$

where the subscript *r* denotes the reduced matrix and:

$$M_r=V^T MV \, , K_r=V^T KV \, , F_r=V^T F \, , L_r=V^T L \tag{4-A}$$

It is worth noting that $y_r(\omega)\approx y(\omega)$. Due to its low dimensionality, the solution to (4) is much faster than the original higher dimensional model. The input and output vectors are the same dimension as (2). Several methods exist to choose *V*. In this work, we choose the projection matrix *V* to be a Krylov subspace in order to provide the moment matching properties [2, 3, 6].

2.1 Model Order Reduction for coupled structural acoustic systems:

For a mutually coupled structural acoustic case, we start off from Cragg's pressure formulation[7]:

$$\begin{pmatrix} Ms & 0 \\ Mfs & Ma \end{pmatrix}\begin{Bmatrix} \ddot{u} \\ \ddot{p} \end{Bmatrix}+\begin{pmatrix} Cs & 0 \\ 0 & Ca \end{pmatrix}\begin{Bmatrix} \dot{u} \\ \dot{p} \end{Bmatrix}+\begin{pmatrix} Ks & Kfs \\ 0 & Ka \end{pmatrix}\begin{Bmatrix} u \\ p \end{Bmatrix}=\begin{Bmatrix} Fs \\ 0 \end{Bmatrix}$$
$$y(t)=L^T\begin{Bmatrix} u \\ p \end{Bmatrix} \tag{5}$$

where, *Ms* is the structural mass matrix, *Mfs* is the coupled mass matrix, *Ma* is the acoustic mass matrix, *Cs* is the structural damping matrix, *Ca* is the acoustic damping matrix, *Ks* is the structural stiffness matrix, *Kfs* is the coupled stiffness matrix, *Ka* is the acoustic stiffness matrix, *Fs* is the structural force vector, *y (t)* the output measurement vector and *u*, *p* are the displacements and pressures at nodal co-ordinates respectively. Ignoring damping for the structure and fluid, the coupled equations in the case of harmonic response analysis become:

$$\left[-\omega^2\begin{pmatrix} Ms & 0 \\ Mfs & Ma \end{pmatrix}+\begin{pmatrix} Ks & Kfs \\ 0 & Ka \end{pmatrix}\right]\begin{Bmatrix} u \\ p \end{Bmatrix}=\begin{Bmatrix} Fs \\ 0 \end{Bmatrix} \tag{6}$$

Constant structural or acoustic damping ratio's can be incorporated into the system matrices of (6) avoiding a direct participation of [C], as it is frequency independent by the definition. Although there exists techniques to reduce system matrices with [C], in this paper, we restrict ourselves to undamped and constant structural damping. A straightforward extension can be made to constant acoustic damping. The finite element software, ANSYS [11] formulates constant damping via the command DMPRAT and MP, DMPR which adds imaginary terms to the stiffness matrix according to the relationship:

$$\beta_c=\frac{2}{\Omega}\xi \tag{6-A}$$

Where, β_c is the constant multiplier applied to structural parts of the coupled stiffness matrix, and Ω is the frequency in rad/s and ξ is the constant damping ratio. This implies that the matrix $[K]$ is complex-valued. In other words, the structural stiffness matrix K_s in Eqn (5), Eqn (6) becomes $K_s+i2\xi K_s$

In this case, the approximation becomes:

$$\begin{Bmatrix} u \\ p \end{Bmatrix}=\{x\}=Vz+\epsilon \tag{7}$$

The transfer function of the system $H(s)=[Y(s)/U(s)]$ using the Laplace transform can be written as:

$$H(s)=L^T(s^2 M_{sa}+sC_{sa}+K_{sa})^{-1}F_{sa} \tag{8}$$

Ignoring damping, and expanding (8) using the Taylor series about $s=0$ results in:

$$H(s)=\sum_{i=0}^{\infty}(-1)^i L^T(K_{sa}^{-1}M_{sa})^i K_{sa}^{-1}F_{sa}s^{2i}=\sum_{i=0}^{\infty}m_i s^{2i} \tag{9}$$

Where $m_i = (-1)^i L^T(K_{sa}{}^{-1}M_{sa})^i K_{sa}{}^{-1}F_{sa}$ $(for\, i = 0,\infty)$ are called the moments of $H(s)$ and,

$M_{sa}=\begin{pmatrix} Ms & 0 \\ Mfs & Ma \end{pmatrix}$, $K_{sa}=\begin{pmatrix} Ks & Kfs \\ 0 & Ka \end{pmatrix}$, $F_{sa}=\begin{Bmatrix} Fs \\ 0 \end{Bmatrix}$.

By matching some of these moments of the higher dimensional system about $s=0$, the reduced order model can be constructed, as it directly relates the input to the output of the system. Theoretically, any expansion point within the frequency range of interest can be used, and a real choice depends on required approximation properties. However, explicitly computing such moments tends to be numerically unstable [3, 4], and it is therefore preferable to attempt to implicitly match these moments via the Arnoldi process. Su and Craig [6], showed that if the projection matrix V is chosen from a Krylov subspace of dimension q,

$$K_q(K^{-1}M, K^{-1}F) = span\{K^{-1}F, (K^{-1}M)K^{-1}F, ...(K^{-1}M)^{q-1}K^{-1}F\} \quad (10)$$

then, the reduced order model matches $q+1$ moments of the higher dimensional model. The block vectors $K^{-1}F$ and $K^{-1}M$ can be interpreted as the static deflection due to the force distribution F, and the static deflection produced by the inertia forces associated with the deflection $K^{-1}F$ respectively.

3 THE ARNOLDI ALGORITHM

To avoid numerical problems while building up the Krylov subspace, an orthogonal basis is constructed for the given subspace. This is done using the Arnoldi algorithm. Given a Krylov subspace $K_q\ (A_1,\ g_1)$, the Arnoldi algorithm finds a set of vectors with norm one which are orthogonal to each other, given by:

$$V^T V = I \quad \text{and} \quad V^T A_1 V = H_q \quad (11)$$

Where $H_q = \Re^{qxq}$ is a block upper Hessenberg matrix and $I_q = \Re^{qxq}$ is the identity matrix. *Figure: 1* describes the implemented algorithm, which is used to generate the Arnoldi vectors for the coupled structural acoustic system. For multiple inputs, the block version of the algorithm can be found in [4]. For the coupled structural acoustic case, we obtain:

$$\begin{aligned} &Colspan(V) = K_q(K_{sa}^{-1}M_{sa}, K_{sa}^{-1}F_{sa}) \\ &V^T K_{sa}^{-1} M_{sa} V = H_q \quad \text{and} \quad V^T V = I \end{aligned} \quad (12)$$

Algorithm: 1:

Input: System Matrices K_{sa},M_{sa}, F_{sa}, L and *n* (number of vectors), expansion poin $s = (\omega_E + \omega_B)/2$
Output: *n* Arnoldi vectors
0. Set $v_1 = g_1$
1. For $i = 1 \rightarrow n, do:$
  1.1 Deflation check: $h_{i,i-1} = \|v_i\|$
  1.2 Normalization: $v_i = [v_i^*] / [h_{i,i-1}]$
  1.3 Generation of next vector: $v_{i+1}^* = A v_1$
  1.4 Orthogonalization with old vectors: for j=1 to i:
    1.4.1 $h_{j,1} = v_j^T v_{i+1}$
    1.4.2 $v_{j+1}^* = v_{i+1}^* - h_{j,i} v_j$
2. Discard resulting H_q, and project $M_{sa}, K_{sa}, F_{sa}, L$ onto V to obtain reduced system matrices $M_{Rsa}, K_{Rsa}, F_{Rsa}, L_{Rsa}$ where the subscript *Rsa* represents the reduced structural acoustic matrices.

Figure 1: The Arnoldi Process [3] [4]

4 NUMERICAL TEST CASE

The first test case is a frame-panel structure which was built to test new modelling techniques. The scaled car cabin is modeled as a simple seven sided structure as shown in *Figure: 2(a)*. The structural model is a frame panel structure coupled with the acoustic cavity. Faces of the acoustic model, other than that of the roof were assumed to be acoustically rigid. The acoustic model was modeled using eight noded acoustic brick elements with one pressure degree of freedom at each node. The thicknesses of the beams and the flat panel are 2.5mm and 1.5mm respectively. The frequency range of interest for the coupled dynamic analysis is 0-300Hz. A unit harmonic force is applied to one of the structural nodes of the lower front member to excite the coupled system as show in *Figure: 2(a)*. The pressure response is computed at two nodes in the fluid domain using the Direct Method in ANSYS and MOR via the Arnoldi process. The noise transfer function (Pressure/Force) at nodes representative of driver and passenger ear location is shown in figures *3(a)* and *3(b)*. All computations described in this paper were performed using a Pentium 3GHz, 2GB RAM machine.

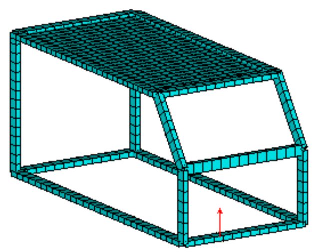

Figure 2(a): Structural FE model

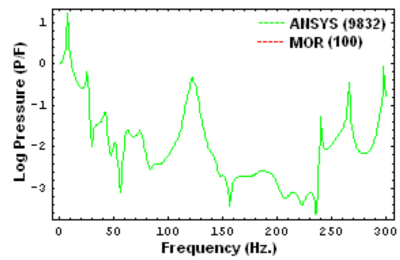

Figure 3(a): Noise Transfer Function

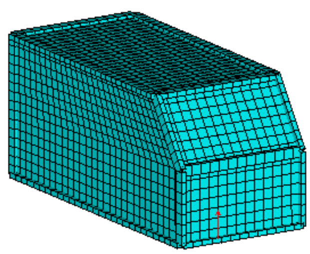

Figure 2 (b) Coupled FE Model

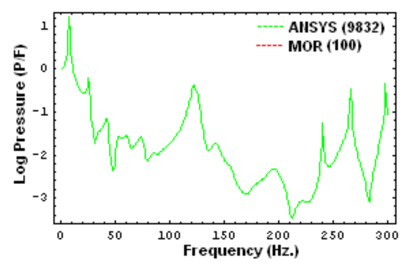

Figure 3(b) Noise Transfer Function

For the reduced order model, the computational time is a combination of four steps (a) Running a partial stationary solution to extract *.FULL file from ANSYS (b) Reading matrices and generating of Arnoldi vectors (c) Projection to second order form and (d) Simulation of the reduced order model. The spilt computational times for the test case are given in *Table: 1*. The reduced order model is set up and solved in Mathematica/MATLAB environment. A comparison of the solution times using MOR and the Direct method in ANSYS are given in *Table: 2.*

Model	Writing FULL file from ANSYS	Read Matrices, Generate Arnoldi Vectors and Project	Reduced Model Simulation	Total: MOR via Arnoldi Process
TC[1]	3 s	492 s [100 Arnoldi Vectors]	22 s	517 s

Table: 1: MOR Split Computational Times; TC[1]: Test Case-1

Model	Elements	Active DOF's	ANSYS Direct	MOR via Arnoldi Process	Time Reduction
TC[1]	6061	9832	7932 s	517 s	93.48%

Table: 2: Computational Times; TC[1]: Test Case-1

5 COUPLED VIBRO-ACOUSTIC OPTIMIZATION

Fiber reinforced composites have generated significant interest among automotive and aerospace manufacturers in the development of structural materials due to their low density, high stiffness and excellent damping characteristics. Additionally, the orthotropic nature of such fiber reinforced composite materials implies that the directional stiffness depends on the orientation of fibers. In this work, the feasibility of reducing interior noise levels through optimal lamination angles of a laminated composite structure via reduced order modelling is demonstrated.

5.1 Optimization Test Case

The optimization test case is coupled model described in *Section:4.* In this case, the roof panel is a symmetric, eight layered Glass fiber reinforced composite structure, with initial lamination angles of $[90/0/90/0]_s$. The lamination angles of the fibers, denoted by θ in this study, take the form of design variables for the optimization problem. The design variables are subject to a lower and upper bounds $0 \le \theta \le 180$ degrees, where 0^0 represents a unidirectional lay-up of the fibers and angles beyond 90^0 represent lay-up in the negative direction. The structural FE model is modeled using a combination of ANSYS SHELL181 and BEAM4 elements, with material properties for the uni-directional composite (for SHELL181) as given in *Table: 3*. A structural damping ratio of 4% is applied to all elements with composite material properties. In addition to this, a constant overall structural damping of 2% is specified for the analysis.

E11=E33 GPa	E22 GPa	G12=G21 GPa	G13 GPa	v12=v13	Density Kg/m^3
28	21	1.39	1.40	0.4	1480

Table:3: Mechanical Properties for the Glass fiber reinforced composite

5.2 Design Optimization Procedure

The reduced order modelling technique outlined in *Section: 2* is incorporated into the optimization process to speed up simulation time, while maintaining the accuracy of the nodal sound pressure values. A general framework of optimization via reduced order modelling is shown in *Figure: 5*. The reduced order model does not allow us to preserve geometry related information, and after changes in the original higher dimensional model, the reduced order model must be regenerated again. Fortunately, the time required to generate a reduced order model is comparable with that for a single frequency evaluation [9]. The number of vectors required to represent the higher dimensional system is calculated using the convergence models presented in [8,13]. In this case, 85 Arnoldi generated vectors were sufficient to represent the higher dimensional system for $\omega=1\text{Hz.}$ and $\omega=300\text{Hz.}$ The non-linear optimization problem can be stated as:

$$\textit{Find a vector of design variables:}\quad \theta=(\theta_1,\theta_2,\theta_3,\ldots\theta_n)$$
$$\textit{Which minimizes the objective function}\quad f(\theta)$$
$$\textit{Subject to lower and upper bounds}\quad \theta_i^{lower}\leqslant\theta_i\leqslant\theta_i^{upper}$$

Numerous possibilities exist to formulate the objective function, and their effectiveness depends on the nature of the coupled problem. For the optimization problem stated above, the objective function is formulated as:

$$f(\theta)=F_{obj}^{\frac{1}{n}};\, F_{obj}=\frac{1}{(\omega_{max}-\omega_{min})}\int_{\omega_{min}}^{\omega_{max}} \vartheta\, p_i(\omega)\, d\,\omega\,,\, where\,,\, \vartheta=\begin{cases}(p_i-p_{ref})^n & for \quad p_i>p_{ref}\\ 0 & for \quad p_i\leq p_{ref}\end{cases} \qquad (13)$$

Where, the function ϑ is a weighting function applied to the nodal sound pressure level (SPL) value p_i. It can be seen that the weighting function depends on reference pressure p_{ref}, determined as 45dB for the current study. For *n=2*, this formulation of objective function (13) results in a frequency averaged root mean square value.

The optimization is carried out using MATLAB GA/PS Toolbox [12] using the Mesh Adaptive Direct Search (MADS) algorithm. Each iteration of MADS is divided into two steps, SEARCH and POLL. The SEARCH step allows the evaluation of the objective function at a finite set of points. Any search strategy can be used, including none. When a SEARCH step fails to improve the objective function value, a POLL step is invoked. In addition to the mesh size parameter, a poll size parameter is defined to ensure that the local exploration of the design variable space is not restricted to a finite set of directions [10]. The set of trial points considered during the POLL step is called a *frame*. Depending on the result of the POLL step, i.e. successful or unsuccessful, the mesh resolution is decreased or increased .A general MADS algorithm is shown in *Figure: 4*. In this work, we chose to evaluate initial trial points in the first iteration of MADS using Latin Hypercube Sampling (LHS).

Algorithm:2:
INITIALIZATION: Define mesh point, mesh size and poll size parameters, set $k \leftarrow 0$
SEARCH AND POLL STEP: Perform SEARCH and POLL steps until an improved mesh
. point is found on mesh.
1) OPTIONAL SEARCH: Evaluate function on a finite subset of trial points on mesh.
2) LOCAL POLL: Evaluate function on computed frame.
PARAMETER UPDATE: Update mesh size and poll size, set $k \leftarrow k+1$ and return to
. SEARCH and POLL steps.

Figure: 4: A simplified MADS Algorithm [10].

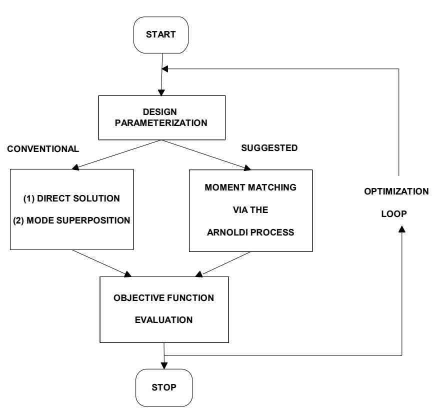

Figure: 5: Optimization Via Moment Matching

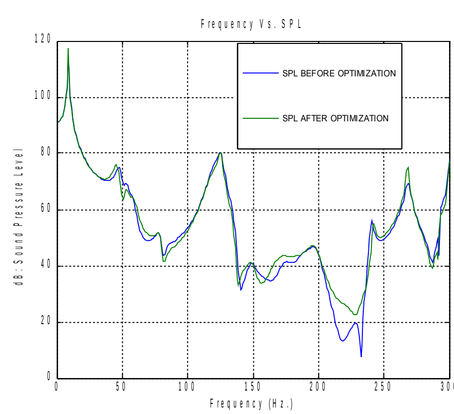

Figure: 6: Sound pressure levels before and after optimization.

5.3 Results

In this paper, symmetric laminates are considered for the optimization study due to structural and manufacturing considerations. From an initial lay up of $[90/0/90/0]_s$ the lamination angles move towards a lay up of $[127/125/35/2]_s$. *Figure: 6* compares the sound pressure level (SPL) before and after optimization. The face sheets of the outer layer of the composite material tend to be moving towards a more even (-53/-55) while the inner lamination angles moves towards an cross-ply orientation (35/2). Sound pressure levels near frequencies ~50Hz, ~85Hz, and ~138Hz and ~245Hz. have decreased by 5.5dB, 2.6dB and 8dB and 9.2dB respectively. In the frequency range of 250-290Hz., peak SPL value has increased from 69.31dB to 74.94dB – an increase of 5.62dB. This is primarily because (a) The optimization is considered over the entire frequency range of 0-300Hz., and an overall decrease in the root mean square SPL value is considered as a successful iteration by the optimizer and (b) the value of $p_{ref}=45\text{dB.}$ is applied to (13). From a computational viewpoint, the use of reduced order modelling significantly decreases the computational time requirements for the optimization process. The time taken by MOR via Arnoldi is shown in *Table: 4*. For the optimization test case, the cost of reduction via Arnoldi is approximately 96% smaller than solving the original higher dimensional ANSYS model. Such smaller cost enables effective application of hybrid search strategies, to search the multi-modal space which usually requires more number of function evaluations. If the optimization had been carried out by the direct method in ANSYS, ~366 hours of computational time would have been required for 166 function evaluations.

Model	Function Evaluations	Initial Stacking Sequence	Final Stacking Sequence	Time MOR	Time ANSYS*	Time Reduction
OPT	166	$[90/0/90/0]_s$	$[127/125/35/2]_s$	49539 s	1316691 s	96.24%

*Table: 4: Optimization Results: OPT: Optimization Test Case.*Estimated time.*

6 SUMMARY

An efficient method to perform coupled structural analysis and optimization via reduced order modelling has been outlined. The basis vectors for matching the coupled system moments are computed by applying the Arnoldi algorithm, which computes the projection vectors spanning the Krylov subspace, to match the maximum number of moments of the system. The moments in the test cases shown are matched at approximately half of the analysis range $s=(\omega_E+\omega_B)/2$. If a Taylor series expansion is considered around a higher frequency, a reduced order model could be obtained with better approximation properties around that frequency range. *Figure: 3(a), 3(b)* indicates that good approximation properties can be obtained by projecting the higher dimensional system to a lower dimension and matching some of the low frequency moments of the system.

In this work, the explicit participation of [C] is avoided by using a complex stiffness approach $K_s(1+i2\xi)$. For a higher dimensional model with [C], a reduced order model via the Second Order Arnoldi (SOAR) process, involving computing orthogonal vectors belonging to the second order Krylov subspaces [14] or transforming the *Equation (6)* to first order form and matching moments via Arnoldi is possible, but a comparison of accuracy and efficiency of such methods is beyond the scope of this current paper. The number of vectors needed to accurately represent the system was 85 for both test case and the optimization test case respectively. Comparing the computational times of the test case and the optimization, it can be seen that the time reduction in the optimization test case is 2.76% higher. In the first test case described in this paper, 100 vectors were generated initially to check for error convergence. However, it is also worth noting that the process of computing the minimum number of required vectors can be completely automated by a user defined error parameter. Lastly, the reduced order modelling framework via the Arnoldi process was incorporated in the structural-acoustic optimization process. The lamination angles of the composite structure took the form of design variables for the optimization problem. As stated earlier, any change in material properties required generation of reduced order model from the higher dimensional model. A general decrease in SPL over the entire frequency range of optimization is evident. Both mode shifting and peak splitting phenomenon's resulting from change in lamination angles were accurately captured by the reduced order model. Finally, it should be noted that MOR via Arnoldi [3,4] would be appropriate only in the low frequency range for vehicle/aerospace structural acoustic applications, and other techniques exists e.g. Energy Finite Element Method (EFEA), Statistical Energy Analysis (SEA), to deal with higher frequencies, where modal density is often high, and the acoustic response is very sensitive to minor structural modifications.

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