MEMS Compact Modeling Meets Model Order Reduction: Examples of the Application of Arnoldi Methods to Microsystem Devices

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ABSTRACT

Modeling and simulation of the behavior of a system consisting of many single devices is an essential requirement for the reduction of design cycles in the development of microsystem applications. Analytic solutions for the describing partial differential equations of each component are only available for simple geometries. For complex geometries, either approximations or numerical methods can be used. However, the numerical treatment of the PDEs of thousands of interconnected single devices with each exhibiting a complex behavior is almost impossible without reduction of the order of unknowns to a lower-dimensional system. We present a fully automatic method to generate a compact model of second-order linear systems based on the Arnoldi process, and provide an example of successfull model order reduction to a gyroscope.

Keywords: Arnoldi process, model order reduction, compact modeling, second order differential equations, butterfly gyroscope

1 INTRODUCTION

For the computational treatment of electronics and microsystems (MEMS[1]), different approaches can be employed. In this section, we review a conventional approach, that is, simplifying a system to an equivalent circuit by hand-made or semiautomatic compact models. We suggest a new way to automate the generation of low-dimensional systems of equations by means of mathematical techniques.

What separates MEMS from purely electronic devices (such as very large scale integration or VLSI transistors and other circuit elements) is that MEMS devices are transducers that convert signals between electronics and all other energy domains. For example, most microgyroscopes and accelerometers found in automobiles are currently produced using MEMS technology. Their coupling functionality results in special requirements for the modelling of MEMS. But also undesired coupling – parasitic effects – need a thorough consideration, since on this small scale mutual influence can become a severe problem.

Thus, the engineers need to simulate the system as a whole. By experience, they are able to define coupling effects between devices, which can be probed at certain terminals. For electrical devices, there is often a natural choice of these terminals, e.g. the emitter, collector and base of a bipolar transistor. However, e.g., for the temperature transport from a computer microchip , this choice is not so obvious.

Once these terminals are identified, the microsystem can be partitioned into a number of devices and energy domains, each coupled by terminals.

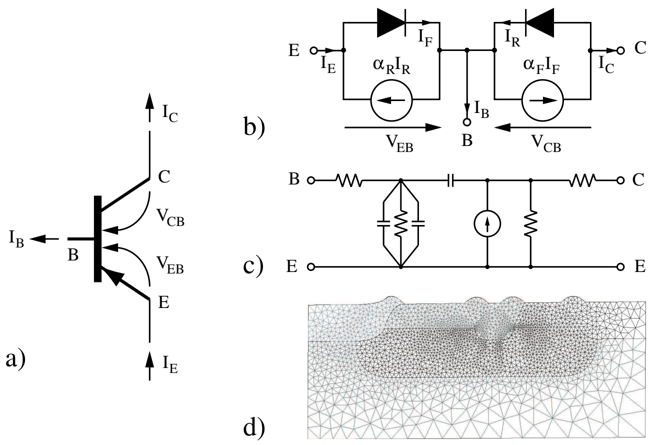

Figure 1: Different modeling approches for an p-n-p transistor. a) Transistor representation for circuit diagram. b) Ebers-Moll compact model of a transistor. c) Compact model for small signal dynamical behaviour analysis. d) Mesh for numerical discretization of PDEs. b) and c) adapted from [1], d) own model (unpublished).

1.1 Compact Modelling vs. Model Order Reduction

In electrical engineering, the common approach is to find a "compact model" of a single device in an analytical form. Whereas there is almost no problem to write down a relationship for simple circuit elements such as resistors and capacitors, the modeling of semiconductor devices was a challenge right from the start. In principle, to accurately describe the transistor operation one should solve the transport PDEs for

[1]We will call all microsystems MEMS, although more functions than only micro-electromechanics are possible.

electrical carriers coupled with a Poisson-Boltzmann equation.

This is possible in analytic form for some special cases. However, as technology develops, the old compact model cannot be applied any more to a newly developed device, and newer models must be employed (see fig. 1).

For MEMS, due to the large number of possible devices, working principles and design freedoms for the engineer, there is no “transistor” device, so that hand-made models are not a viable solution for the long term.

On the other hand, automatic model order reduction (MOR) aims at providing reduced models only with minimal intervention by the designer. The goal is to provide a software which - based on a spatial discretization of the PDE, e.g. by the finite element method - is capable to return ODEs with a far lower number of state variables than the previous discretized system without sacrificing too much acuracy. These ODEs can then be used in SPICE-like simulators, allowing for system simulations in acceptable time.

The designer does not need to worry about the details of the reduction process, and the software should be robust enough for use in industrial applications. Model order reduction thus provides ”Compact Modeling on Demand”.

1.2 State of the art and the future of automatic model reduction

At present, MOR of first and second order linear ordinary differential equation can be considered as solved. These equations occur in a large number of cases in microsystem engineering. In electronic circuits, during a small signal analysis, linearization and replacement by a simpler equivalent circuit is also often possible. Very often possible nonlinearities are mostly suppressed by a suitable feedback circuit, and so the assumption of a linear system is quite valid. MOR can be an important part for the design of those components.

For some cases like bilinear or quadratic nonlinearities (occuring e.g. in fluid dynamics), or for nonlinearities occuring near a given state trajectory, recently solutions were presented [2], [3]. However, to be able to compete with sophisticated nonlinear transistor compact models, more research is certainly needed. At the moment, the usefulness lies especially in coupling multiphysics simulations with highly depelopped compact circuitry models.

2 ARNOLDI PROCESS FOR SECOND ORDER SYSTEMS

The first step for finding a reduced order model is to formulate a discretized version of the system’s PDE. For example, considering the force equilibrium for a linear time invariant elastic system, we obtain the linear system

$$\mathbf{M}\ddot{\boldsymbol{x}}(t) + \mathbf{C}\dot{\boldsymbol{x}}(t) + \mathbf{K}\boldsymbol{x}(t) = \mathbf{B}\boldsymbol{u}(t), \qquad (1)$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are called the mass, damping, and stiffness matrix and $\mathbf{B}$ is the scattering matrix to distribute the inputs $\boldsymbol{u}(t)$ on the domain.

Often the engineer is only interested in the solutions of a few degrees of freedom or linear combinations thereof. A selector matrix $\mathbf{L}^T$ yields the output vector $\boldsymbol{y} = \mathbf{L}^T\boldsymbol{x}$.

After Laplace-transformation of (1) the transfer function $\boldsymbol{H}(s) = \mathcal{L}(\boldsymbol{Y}(s))/\mathcal{L}(\boldsymbol{U}(s))$ can be written as

$$\boldsymbol{H}(s) = \mathbf{L}^T \left(s^2\mathbf{M} + s\mathbf{C} + \mathbf{K}\right)^{-1} \mathbf{B}. \qquad (2)$$

The goal of MOR is to find a new system of equations

$$\mathbf{M}_r\ddot{\boldsymbol{z}}(t) + \mathbf{C}_r\dot{\boldsymbol{z}}(t) + \mathbf{K}_r\boldsymbol{z}(t) = \mathbf{B}_r\boldsymbol{u}(t), \quad \boldsymbol{y} = \mathbf{L}_r^T\boldsymbol{z} \qquad (3)$$

with a lower number of equations n_r and a low dimensional state vector $\boldsymbol{z}$ such that the transfer function is near to the transfer function of the original system.

A number of mathematical procedures are available to achieve this, the most popular probably projection algorithms that replace $\boldsymbol{x}$ by a lower dimensional state vector $\boldsymbol{z}$ such that $\boldsymbol{x} = \mathbf{V}\boldsymbol{z}$. Our approach is based on the Arnoldi process. This algorithm returns a projected system whose first terms of the Taylor series of the transfer function match those of the full system. Details are presented elsewhere [5] (and references in there).

3 THE BUTTERFLY GYRO

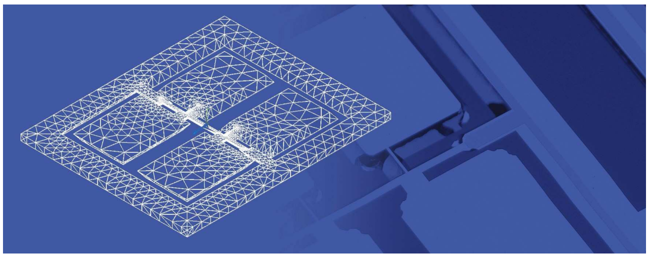

Figure 2: Finite element mesh of the gyro with a background photograph of the gyro wafer pre-bonding.

The *Butterfly* gyro is developed at the Imego Institute in an ongoing project with Saab Bofors Dynamics AB. The *Butterfly* is a vibrating micro-mechanical gyro that has sufficient theoretical performance characteristics to make it a promising candidate for use in inertial navigation applications. The goal of the current project is to develop a micro unit for inertial navigation that can be commercialized in the high-end segment of the rate sensor market. This project has reached the final stage of a three-year phase where the development and research efforts have ranged from model based signal processing, via electronics packaging to design and prototype manufacturing of the sensor element. The project has also included the manufacturing of an ASIC, named μSIC, that has been especially designed for the sensor (fig. 4).

The gyro chip consists of a three-layer silicon wafer stack, in which the middle layer contains the sensor element. The sensor consists of two wing pairs that are connected to a common frame by a set of beam elements (figure 2); this is the reason the gyro is called the *Butterfly*. Since the structure is manufactured using an anisotropic wet-etch process, the connecting beams are slanted. This makes it possible to keep all electrodes, both for capacitive excitation and detection, confined to one layer beneath the two wing pairs. The excitation electrodes are the smaller dashed areas shown in fig. 3. The detection electrodes correspond to the four larger ones.

By applying DC-biased AC-voltages to the four pairs of small electrodes, the wings are forced to vibrate in anti-phase in the wafer plane. This is the excitation mode. As the structure rotates about the axis of sensitivity (fig. 3), each of the masses will be affected by a Coriolis acceleration. This acceleration can be represented as an inertial force that is applied at right angles with the external angular velocity and the direction of motion of the mass. The Coriolis force induces an anti-phase motion of the wings out of the wafer plane. This is the detection mode. The external angular velocity can be related to the amplitude of the detection mode, which is measured via the large electrodes.

When planning for and making decisions on future improvements of the *Butterfly*, it is of importance to improve the efficiency of the gyro simulations. Repeated analyses of the sensor structure have to be conducted with respect to a number of important issues. Examples of such are sensitivity to shock, linear and angular vibration sensitivity, reaction to large rates and/or acceleration, different types of excitation load cases and the effect of force-feedback.

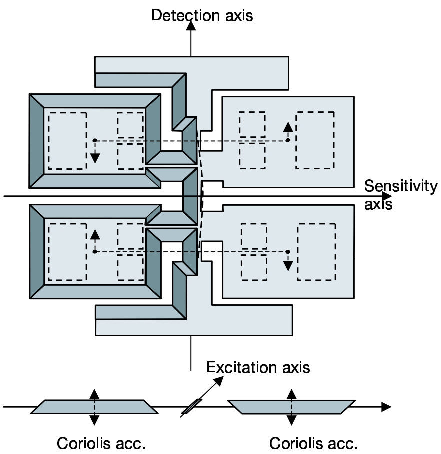

Figure 3: Schematic layout of the *Butterfly* design.

The use of model order reduction indeed decreases runtimes for repeated simulations. Moreover, the reduction technique enables a transformation of the FE representation of the gyro into a state space equivalent formulation. This will prove helpful in testing the model based Kalman signal processing algorithms that are being designed for the *Butterfly* gyro.

Figure 4: The *Butterfly* and μSIC mounted together.

4 RESULTS

We reduced an *ANSYS* model of the *Butterfly* gyroscope from 17361 degrees of freedom to models with different lower orders. Due to the properties of the Arnoldi process, all reductions with a lower order are contained in a higher order reduced model, model, so it is sufficient to perform the reduction for the largest order desired. In this case, we reduced the model to 40 degrees of freedom.

The time to create this model is about the same as the calculation of a single timestep in *ANSYS*.

All calculations for the full model were performed in *ANSYS*. The reduction process itself is performed by an external C++ program, which operates on the *ANSYS* .emat files and outputs the reduces matrices as well as projection matrices. The postprocessing is done im *Mathematica*.

4.1 Time domain

Figure 5 shows a comparison of the transient behavior for the full model and some examples of a reduced model. We see that while order 5 is not good enough (fig. 5a), the reduced model of order 10 is already very good (fig. 5b,c).

4.2 Frequency domain

Figure 6 shows a comparison of the transfer functions. While the reduced models of order 5 up to 15 show considerable deviations for the low frequency range, the model with order 20 shows a perfect match for a larger extend. The order 40 model is even closer for higher frequencies, though this is not so important for the gyroscope. For a step input with its large portion of low frequencies, and the timescale considered in fig. 5, order 10 yields already very satisfying results.

5 DISCUSSION

The exceptionally good results were also demonstrated for other energy domains. Every linear problem in *ANSYS* can be model order recuced this way. It is also possible to extend the tool to other simulation packages as long as the system matrices can be recovered. Various examples linear first and second order systems were succesfully reduced, thereby showing distinct advantage over commercially available methods:

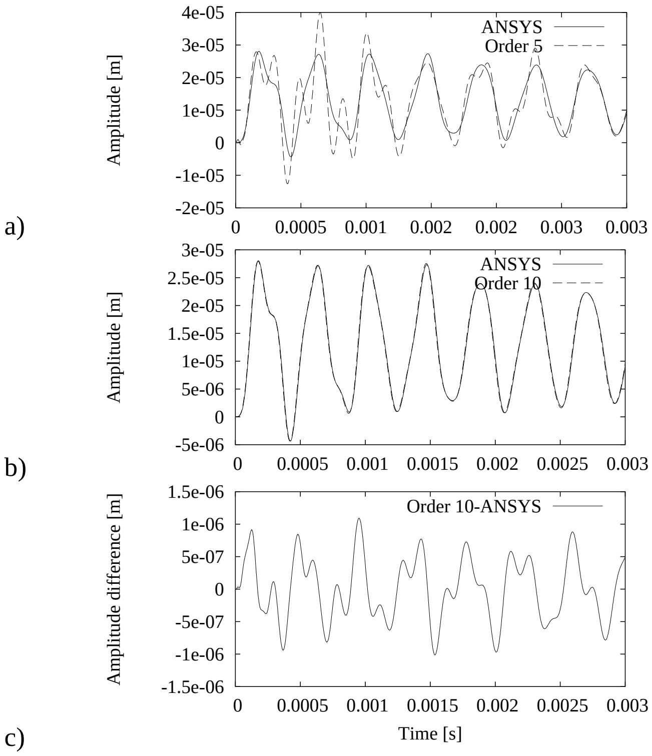

Figure 5: Comparison of transient behavior for full and reduced models: a) model order 5 vs. *ANSYS*, b) model order 10, c) difference between model order 10 and *ANSYS*.

- First order thermal and electro-thermal systems [6]
- Second order mechanical systems
- Piezoelectric actuation of a surface acoustic waves device
- Acoustic simulations
- Electromagnetic systems.

Some results for these systems are published [5], [6], thereby showing distinct advantage over e.g. the Guyan method implemented in *ANSYS*, others are in preparation.

6 CONCLUSIONS

Model order reduction techniques provide a valuable tool for the designer of coupled multiphysics system. Especially for applications with a large number of similar devices, as often encountered in microsystem applications, the method facilitates a low time to market and a increase of design quality due to the possibility to simulate whole systems. As an example, we successfully reduced the order of an ANSYS model by four orders of magnitude. Even for a low number of degrees of freedom, low frequency transient curves showed an excellent match.

This method works perfectly for first or second order linear time invariant systems. But also for nonlinear or time variant systems, research results are coming and raise hope

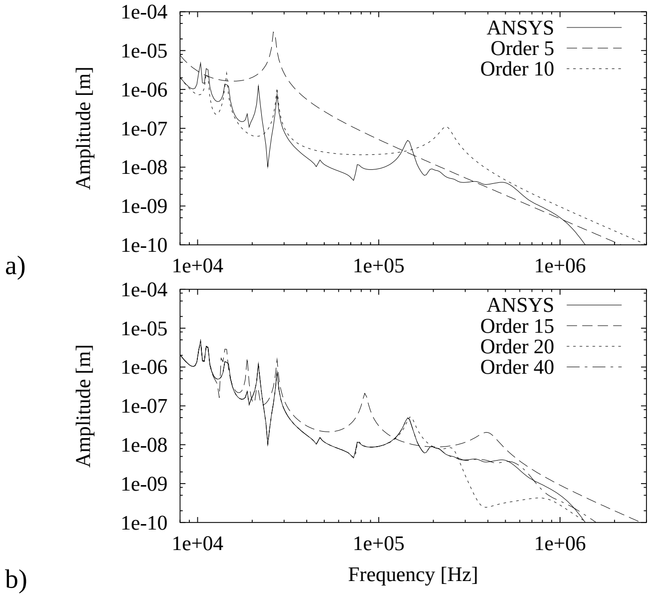

Figure 6: Comparison of transfer functions of the full and reduced models.

to be able to simulate nonlinear elements like transistors in the future.

7 ACKNOWLEDGMENTS

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