Parametric Model Reduction to Generate Boundary Condition Independent Compact Thermal Model

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ABSTRACT

An application of formal model reduction to generate a boundary condition independent compact thermal model is discussed. A new method to find a low-dimensional basis that preserves the convection coefficient as a parameter is presented. Numerical results show that the method allows the convection coefficient to change from 1 to 10^9 while keeping the accuracy of the reduced model to within a few percents.

1. INTRODUCTION

In the development of integrated circuits and microsystems [1][2], thermal management is always essential. There are several electro-thermal and thermo-electric coupling but probably the biggest concern is about the Joule heating effect, which generates heat during conduction of the electric current through a resistor. In an integrated circuit, one has to remove the generated heat to keep the board temperature within acceptable limits. In microsystems, the Joule heating is often employed to keep a designated part (hotplate) at a given elevated temperature. In any case, the right temperature regime is crucial for the correct system functioning and its reliability.

The finite element method allows us to make accurate models to describe the heat transfer but their high dimensionality prevents engineers to employ them during system-level simulation. Hence, an important practical question is how one can make accurate but low-dimensional thermal models.

The two European projects, DELHPI and PROFIT have addressed this need: to produce an accurate but a small thermal model of a chip [3][4]. The DELHPI project has identified number of requirements for a compact thermal model among which one of the most important is that the compact model must be boundary condition independent [5][6][7]. This means that a chip producer does not know conditions under which the chip will be used and hence the chip compact thermal model must allow an engineer to research on how the change in the environment influences the chip temperature. The chip benchmarks representing boundary condition independent requirements are described in [8]. Related discussions are also in Refs [9][10]. The goal of the PROFIT project was to extend the methodology to transient compact thermal models by using methods from [11][12].

The development of a compact thermal model is still a hot topic [13][14] without being completely solved. Recently, modern methods of model order reduction have been successfully applied to automatically generate a compact thermal model [15]-[18], however they do not meet the criterion of boundary condition independence. In the present work, we make a mathematical statement of a problem in question (parametric model reduction) and we suggest the first formal solution that proved to be very efficient. The remaining of the paper is organized as follows. In Section 2, we introduce the parametric model order reduction method and then we show numerical results in Section 3. At the end, we give our conclusions.

2. PARAMETRIC MODEL REDUCTION

In this section, we first describe the mathematical problem that appears after the application of the finite element method. Then, we briefly review conventional model order reduction [16-18]. Finally, we propose the parametric model order reduction technique.

2.1 Discretizing the heat transfer equation with the convection boundary conditions

The partial differential heat transfer equation

$$\nabla \cdot (\kappa \nabla T) + Q - \rho C_p \frac{\partial T}{\partial t} = 0 \quad (1)$$

together with the convection boundary conditions

$$q = h(T_g - T_s) \quad (2)$$

can be discretized by a finite element method. It gives

$$C\frac{dx(t)}{dt}+\left(G+\sum_i k_i D_i\right)x(t)=B$$
$$y(t)=Ex(t) \quad (3)$$

where $C, G, D_i \in R^{n\times n}$ respectively, D_i is diagonal. $x\in R^n$ is the unknown vector. $E\in R^{m\times n}$ represents the relationship between *x* and the output response *y*. *k* is a film coefficient describing the heat flow between the device and neighboring fluid phase. Its value can change significantly with the changing of the device environment. In [8], its numerical values change from 1 to 10^9. Convection boundary conditions at different surfaces can have different k_i. In the present paper we limit ourselves to the case, when k_i is uniform, that is, to the following equation

$$C\frac{dx(t)}{dt}+\left(G+kD\right)x(t)=B$$
$$y(t)=Ex(t) \quad (4)$$

The number of nodes during the discretization can routinely reach 100 000, especially in the case of three-dimensional models. This means that the order *n* of the system matrix *C, G, D* is very large which makes the use of the system (4) infeasible during system-level simulation.

Model order reduction techniques have been proved to be promising for fast simulation of large-scale ordinary differential equations in circuit systems and micro-electronic-mechanical systems (MEMS) [16]-[18]. Yet, a conventional model order reduction method can only deal with system (4) when *k* is fixed. In the next subsection, we will introduce the conventional way of model order reduction on system (4) and show its limitation. We propose our parameter independent method in section 2.3.

2.2 Conventional model order reduction

The basic idea of conventional model order reduction method [16]-[18] is to find a Padé approximation of the transfer function of the original system. The Arnoldi process allows us to achieve it by numerically efficient and stable means. The reduced order time domain system can be derived through the projection matrix.

The Laplace transformation converts Eq (4) to the frequency domain

$$sCX(s)+\left(G+kD\right)X(s)=BU(s)$$
$$Y(s)=EX(s) \quad (5)$$

From Eq (5), we can write the transfer function as follows

$$H(s)=Y(s)/U(s)=E(sC+kD+G)^{-1}B$$

In order to apply the conventional model reduction, we must fix the convection coefficient so that $k=k_0$ is a constant number. Now *H(s)* can be expanded into series around an expansion point s_0 such that $s=s_0+\sigma$, as follows

$$H(s)=E((s_0+\sigma)C+k_0D+G)^{-1}B$$
$$=E\sum_{i=0}^{\infty}\widetilde{M}_i\sigma^i$$

where

$$E\widetilde{M}_i=E[-(s_0C+k_0D+G)^{-1}C]^i(s_0C+k_0D+G)^{-1}B$$

is the so called moments of *H(s)*. The orthogonal projection matrix $\widetilde{V}\in R^{n\times q}$ is constructed in the following way,

$$spancolumn\{\widetilde{V}\}=span\{\widetilde{M}_0,\widetilde{M}_1,\widetilde{M}_2,\cdots\widetilde{M}_q\} \quad (6)$$

Numerically $\widetilde{V}$ is obtained by the Arnoldi process. Then the reduced system in time domain can be obtained by projecting the original unknown vector *x* into the subspace spanned by $\widetilde{V}$, i.e. with approximation $x\approx\widetilde{V}z$:

$$\widetilde{V}^T C\widetilde{V}\frac{dz(t)}{dt}+\widetilde{V}^T\left(G+k_0D\right)\widetilde{V}z(t)=\widetilde{V}^T B$$
$$\widetilde{y}(t)=E\widetilde{V}z(t) \quad (7)$$

Usually $q<<n$, the order of the matrices in (7) is of much smaller size than that in the original system (4), which makes the simulation of (7) much faster. q is the number of moments included in the projection matrix in (6), which safeguards in theory that *q* moments can be matched by the transfer function of the reduced system. The more moments are matched the more accurate by the approximation $x\approx\widetilde{V}z$. If the approximation can reach a satisfactory accuracy, then the reduced model will be a good substitution of (4) when *k* is a constant. Unfortunately, the reduced model (7) can be used for given *k* only. If we change it, the model reduction process should be repeated again.

2.3 Derivation of parametric reduced model

Mathematically speaking, the model reduction scheme should be modified to allow us to keep *k* as a parameter in the reduced model. In other words, boundary condition independent features require parametric model reduction.

In general, parametric model reduction may not be possible. However, in our case, the parameter enters Eq (4) in such a simple form that the projection mechanism described in the previous subsection can preserve it.

Let us show this. The reduced model can then be derived with the approximation $x \approx V\hat{x}$ in original system (4)

$$\begin{aligned} & CV\frac{d\hat{x}(t)}{dt} + G\hat{x}(t) + kDV\hat{x}(t) = B \\ & \hat{y}(t) = EV\hat{x}(t) \end{aligned} \tag{8}$$

and multiply V^T on both sides

$$\begin{aligned} & \hat{C}\frac{d\hat{x}(t)}{dt} + \hat{G}\hat{x}(t) + k\hat{D}V\hat{x}(t) = \hat{B} \\ & \hat{y}(t) = \hat{E}\hat{x}(t) \end{aligned} \tag{9}$$

where

$$\hat{C} = V^T CV, \hat{G} = V^T GV, \hat{D} = V^T DV, \hat{E} = V^T EV$$

Note that Eq (9) can work for any *k* and the problem is how to find such a *V* that gives small error for any *k*. In the present work, we define the relative error between the original and reduced systems as follows

$$error = [\sum_{i=1}^{n} (y_i - \hat{y}_i)^2]^{1/2} / [\sum_{i=1}^{n} y_i^2] \tag{10}$$

where $y = (y_1, y_2, \cdots y_n)^T$ is the solution of system (4) by direct numerical simulation. $\hat{y} = (\hat{y}_1, \hat{y}_2, \cdots \hat{y}_n)^T$ is the approximate solution by the reduced model. If we use $\widetilde{V}$ from conventional model reduction for some k_0, then this does not work well. In principle, one can generate several reduced models for different k_0 but one needs too many of them.

After several tries, we have discovered that the subspace generated by

$$\begin{aligned} & spancolumn\{V_\mu\} = \\ & span\{G^{-1}B, (G^{-1}(C + \mu D)G^{-1}B, \\ & (G^{-1}(C + \mu D))^2 G^{-1}B, \cdots, \\ & (G^{-1}(C + \mu D))^j G^{-1}B\} \end{aligned} \tag{11}$$

produces the required basis.

The problem is that *C* and *D* have different physical units and this should be corrected by the introduction of a factor μ to *D* as shown above and the choice of the factor is to be researched.

We will demonstrate numerical results supporting our claim in the next section.

3. NUMERICAL RESULTS

In this section, we present numerical results to further confirm the proposed boundary independent reduced model in section 2.3 with an example of a microthruster unit shown in Fig. 1. More detail about the device is given elsewhere [17]. Actually, it is quite similar to a chip model: it has a heat source and the generated heat dissipates through the device to the surrounding. Its discretization has produced a system of ordinary differential equation (4) with the dimension of the state vector of 4257.

The "error" presented in the figures is defined by Eq (10) as the average relative error between the original and reduced systems. The order of the original system in (4) is *n*=4257, i.e. $y \in R^n$, and that of the reduced system in (7) or (9) is *q*=20, i.e. $z \in R^q, \hat{y} \in R^q$.

In Fig. 2, the temperature response at the middle of the heater is shown with different values of *k* in order to show that output of the system vary significantly when the parameter *k* changes. Note the logarithmic scale.

Fig. 3 presents errors for reduced models computed by the conventional method. They are very small provided the reduced model is generated for each *k* individually (see the solid line). If we use projection matrix in Eq (6) for particular k_0 to generate the reduced model in (9), the error rapidly grows when we change *k* in the reduced model. In other words, the basis generated for a particular k_0 can be used just for a small range of *k*.

In Fig. 4, the error for project basis computed by Eq (11) is plotted. The dashed line is the error as a function of *k* for a single reduced model with μ=1. The error reaches 5 % that actually is considered reasonable by the DELPHI authors[6][7]. This can be further reduced to 1% if the original system in (4) is approximated by three reduced models with $\mu = 1$, 1000, 1000000.

4. CONCLUSION

The discretization of partial differential equations for some engineering problems results in large-scale ordinary differential systems, which will make the simulation too slow for system-level simulation.

We have found a projection subspace that allows us to generate a reduced model for wide range of *k*.

The research was done in an empiric fashion. It is expected that some theoretical explanation can be offered in future work.

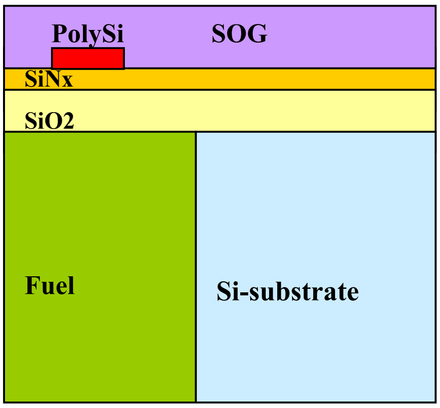

Fig. 1. A 2D-axisymmetrical model of the microthruster unit (not scaled). The axis of the symmetry on the left side. A heater is shown by a red spot.

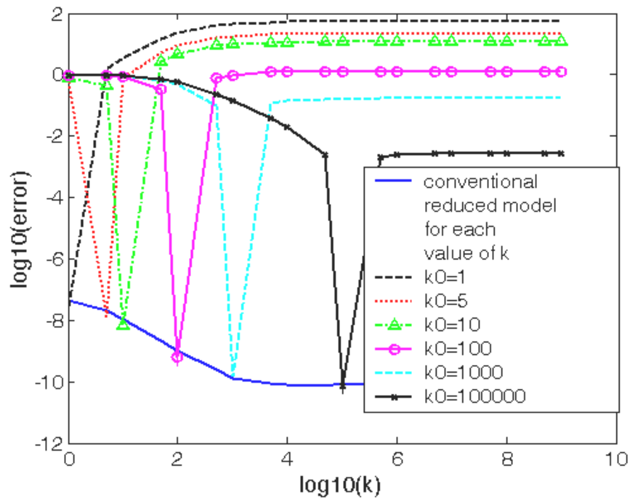

Fig. 3 Error of proposed reduced model

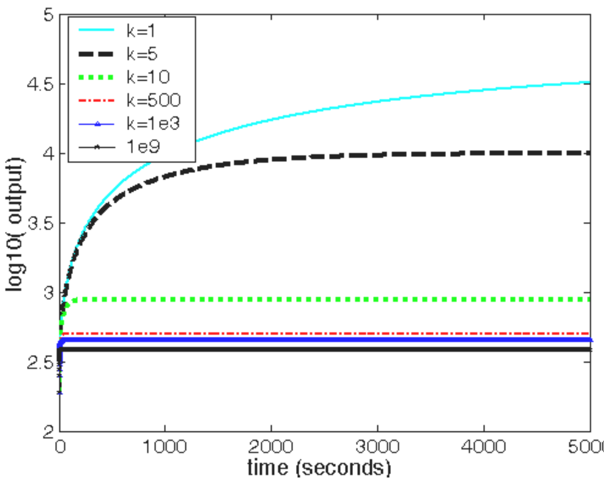

Fig.2.a Output at node 1 with different values of *k*

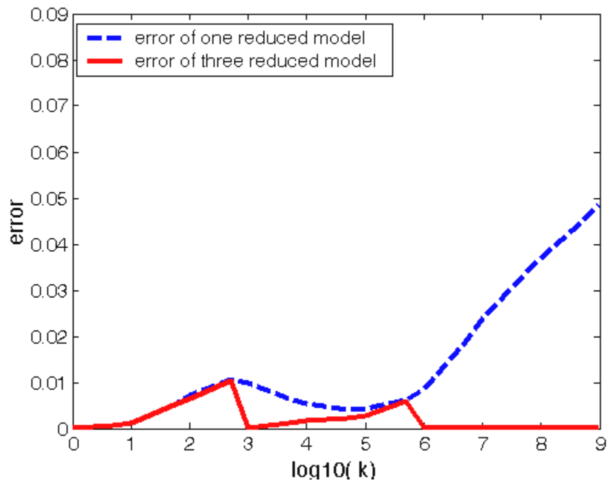

Fig.4 Error by conventional model reduction method

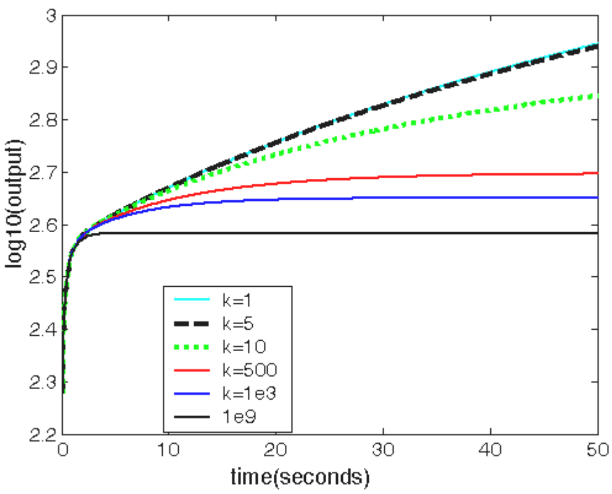

Fig.2.b Enlarged part of Fig.2.a

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