Model Order Reduction for the extraction of small signal equivalent circuit models of RF-MEMS

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***Abstract*—A reduced order model for the small signal analysis of micromechanical structures (MEMS) has been extracted by applying model order reduction (MOR) to their finite element models. The low-order model conserves the accuracy belonging to the finite element method, while drastically reducing the computational time. Moreover, it gives a description of the device's terminal behaviour and can therefore be employed for circuit and system level simulations of MEMS devices.**

Keywords—MOR, electromechanical coupling, harmonic pre-stressed analysis

I. Introduction

Micromechanical structures with electrostatic actuation for radio-frequency (RF) applications have been recently developed. Due to their low power consumption, small dimensions, and good performance compared with their conventional counterparts, these devices are considered as promising building blocks for future reconfigurable transceiver front-end architectures [1].

Because of the electromechanical coupling of MEMS devices, their modeling and design are non trivial and pose specific simulation requirements. Both mechanical and electrical behaviour of devices needs to be accurately described to allow the device model to interface with external electronic circuitry. Moreover, in order to enable an iterative design optimization process, both at device and system level, the computational complexity of simulation needs to be minimized [2]. At present, the common approach to fulfill these requirements is the extraction of a compact equivalent circuit model of the device, whose parameters are computed either analytically or with single energy domain finite element simulations (FEM) [3]. Finite element simulators offer the possibility to solve coupled-field harmonic pre-stressed simulation, but the computational cost of the simulation is very high. This precludes the employment of the full finite element model in system level simulations. Several methods have been developed to build macromodels which contain the same information of a dynamic FEM simulations, but can be used in a circuit or system level simulation environment [4]–[6]. In this work a new approach is presented which focuses on the creation of macromodels for small signal analysis of electromechanical structures. In particular, harmonic pre-stressed analysis is treated that is important for the characterization of frequency-selective devices. The model order reduction technique is applied to a finite element electromechanical model to extract a low order model which preserves modeling accuracy and enables simulation speed up and direct interfacing with electrical components. The adopted procedure is described and applied to the simulation of a micromechanical ring resonator.

II. Simulation approach

The finite element software ANSYS® is used to build and mesh the micromechanical device model. In order to allow for both harmonic simulation and MOR application, a strong coupling between the electrical and the mechanical problem is needed. This is achieved using lumped transducer elements (TRANS126) available in ANSYS [7], which model the capacitance between movable parts of the device as a function of their distance. A non-linear analysis allows to compute the bias point of the device around which the small-signal harmonic analysis will be performed. The linearized stiffness matrix is computed at this bias point, thus taking into account both electrical and mechanical effects of the static voltage applied to the device. Small-signal sinusoidal voltages are then applied and the super stiffness matrix $\overline{\mathrm{K}}$ and load vector for harmonic simulation are computed for two angular frequency values. These are used for the extraction of the system of element matrices describing the device considering that, for harmonic simulation:

$$\overline{\mathrm{K}} = (\mathrm{K} - \omega^2 \mathrm{M} + j\omega \mathrm{E}) \tag{1}$$

where K, M and E are respectively the stiffness, mass and damping real matrices, ω is the angular frequency and $j = \sqrt{-1}$. In order to obtain a compact model for the employment in circuit simulation a further step is required that enables the current calculation from the computed d.o.f. The current flowing in each transducer element is a function of the displacement x_{ij},the voltage v_{ij} and the capacitance C between its nodes i and j. After linearization to the first order around the bias point (v_b, x_b) and transformation in phasor notation, the expression of the current assumes the form:

$$I = j\omega \left(\frac{\partial C\,(v_b, x_b)}{\partial x} v_b X_{ij} + C\,(v_b, x_b)\, V_{ij} \right) \tag{2}$$

where I, X_{ij}, and V_{ij} are the Steinmetz transforms of i, x_{ij} and v_{ij} respectively. Coefficients needed for the calculation of current are stored in the rows of the E matrix corresponding to voltage d.o.f. With simple matrix operations on matrix E an output matrix C is built which allows

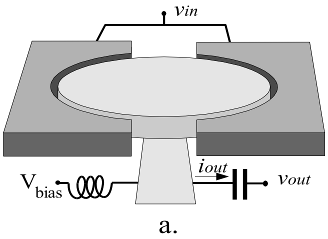

Fig. 1. Measurement setup of a disk resonator in a one-port (a.) and two-port (b.) configuration.

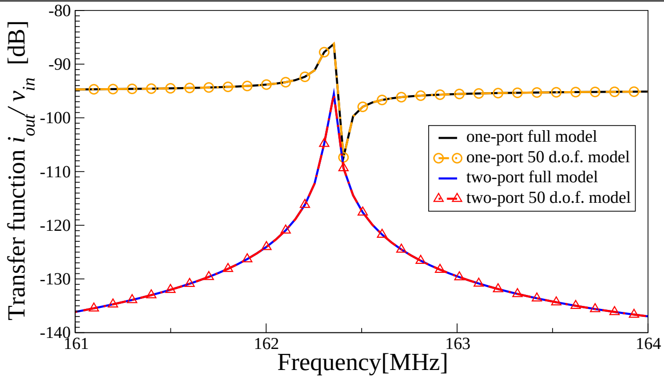

Fig. 3. Transmission spectrum of the disk resonator obtained with full model and reduced model simulation, for the two configuration tested.

for the computation of the current at the nodes of interest. The system $\{M, K, E, B, C\}$ is then reduced applying the tool *mor4ansys*, which performs moment-matching model order reduction via the Arnoldi algorithm [8]. With a proper choice of the current and voltage outputs a behavioral low-order model is obtained, which can eventually be implemented in hardware description language (HDL).

III. Results

The described approach has been used to simulate the response of the radial mode disk resonator described in [9]. The device was operated both in a one-port and a two-port configuration, using the setups of Fig. 1 and grounding the output. Both the full 8000 d.o.f. finite element model and a reduced 50 d.o.f. model have been simulated. Results obtained with the two models are compared in Fig. 2 and 3. Fig. 2 shows the amplitude of the displacement of the edge of the disk with respect to the static position. In Fig. 3 the transmission spectrum i_{out}/v_{in} is plotted. The relative error committed with the low-order model remains less than 1%, so that in the plots the curves cannot be distinguished.

IV. Conclusion

Model order reduction has been proven to be a useful tool for the creation of accurate compact electrical models for harmonic analysis of microelectromechanical devices, which can be used in circuit and system level simulations. The models extracted are specific for the geometry and material considered as well as the applied bias voltage. A parametric model would enhance design optimization both at device and system level. Nevertheless, the extraction process is not very time consuming. It can therefore be repeated for several values of the design parameters, if not for fine design tuning, to study their influence on device and system behaviour. The approach demonstrated can be extended to the treatment of transient small signal analysis.

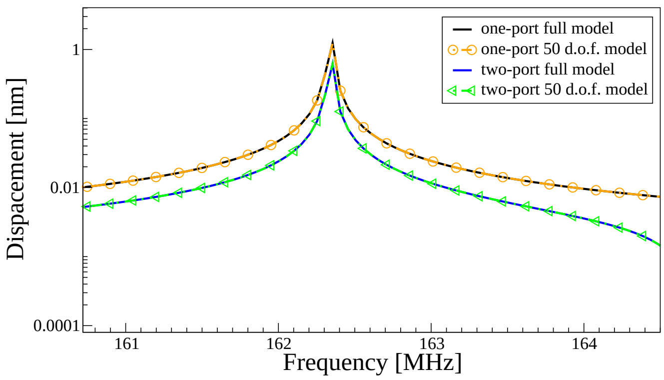

Fig. 2. Amplitude of the displacement of the edge of the disk resonator obtained with full model and reduced model simulation, for the two configuration tested.

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