Automatic Order Reduction for Finite Element Models

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Abstract

In the process of physical modelling of microsystems operating on various energy domains, the engineer is used to apply Finite Element techniques for the discrete representation of the functionality of the device under investigation in a simulation environment. There are many commercial products that help the engineer in performing this task. The common feature of all these simulation tools is that the discrete representation consists of a system of ordinary differential equations. The dimension of this system is directly connected to the number of degrees of freedom for the respective problem. For a spatial displacement field, e.g., the degrees of freedom are three times the number of discretization nodes. The higher the requirements for precision of the simulation results, the more discretization nodes are usually introduced. Nevertheless, the results the engineer will use are in most cases of low dimensional order. In other words, the characteristic features of the required functionality of the device under developement are well represented in low dimensional subspace of the entire solution space of a very fine Finite Element model. Moreover, the requirement for system behaviour simulation makes it impossible to couple large-scale

models for this task. Therefore there is a demanding need for reducing the dimension of the mathematical representation of sub-systems. An approach to this task will be presented in this work, together with the modeling of an RF-microswitch as an example.

1   Introduction

Model reduction of linear large-scale systems is already quite an established area [1, 2, 3]. In this contribution we consider a moment-matching model reduction via the Arnoldi algorithm directly applied to ANSYS linear finite element models. We discuss the application of our tool *mor4ansys* [4] to a structural mechanical problem with a second order linear differential equation. Its successful application to the first order case of electro-thermal modeling has already been demonstrated elsewhere [5, 6].

After a model is built and meshed in ANSYS, an ODE system is obtained. The PDE for an elastic body reads

$$\boldsymbol{f}_I b(\boldsymbol{x}, t) + \boldsymbol{f}_D b(\boldsymbol{x}, t) + \boldsymbol{f}_S b(\boldsymbol{x}, t) = \mathbf{b}(\boldsymbol{x})\boldsymbol{u}(t) \qquad (1)$$

where $\boldsymbol{f}_I$ is the inertial force, $\boldsymbol{f}_D$ is the damping force, and $\boldsymbol{f}_S$ is the elastic force. $\mathbf{b}\boldsymbol{u}(t)$ is an external force depending on the user's input and varying in time [7]. The *scatter matrix* $\mathbf{b}$ distributes the inputs $\boldsymbol{u}$ to the domain. By discretization with n spatial degrees of freedom x_i, $1 \leq i \leq n$, commercial FE-programs transform the PDE to n ordinary differential equations,

$$\boldsymbol{f}_S = \mathbf{K}\boldsymbol{x}, \qquad \boldsymbol{f}_D = \mathbf{C}\dot{\boldsymbol{x}}, \qquad \boldsymbol{f}_I = \mathbf{M}\ddot{\boldsymbol{x}} \qquad (2)$$

$$\implies \quad \mathbf{M}\ddot{\boldsymbol{x}}(t) + \mathbf{C}\dot{\boldsymbol{x}}(t) + \mathbf{K}\boldsymbol{x}(t) = \mathbf{B}\boldsymbol{u}(t), \qquad (3)$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the system matrices, $\mathbf{B}\boldsymbol{u}(t)$ is the load vector, and $\boldsymbol{x}$ is a vector with unknown degree of freedoms, its dimension being routinely from 10 000 to 1000 000. The outputs of the system can in principle be an arbitrary linear combination of states

$$\boldsymbol{y} = \mathbf{L}^T \boldsymbol{x}, \qquad (4)$$

but usually $\mathbf{L}$ is only used to pick certain degrees of freedom.

The information from the FE-program is transferred as a file with element matrices and lists for Dirichlet boundary conditions, nodal forces and output

degrees of freedoms. The developed software uses these files as input data and produces a reduced model by means of the Arnoldi algorithm.

The user chooses a maximum order m for the reduced model. The iterative nature of the Arnoldi algorithm makes all possible reduced models with dimensions ranging from 1 to m available. The remaining postprocessing task is the solution of a reduced ODE system, as well as to calculate its transfer function. It is worth mentioning that this can be done in any environment, since the reduced model is stored as an ASCII file.

A conventional approach to model reduction is to find a low-dimensional subspace $\mathbf{V}$

$$\boldsymbol{x} = \mathbf{V}\boldsymbol{z} + \boldsymbol{\varepsilon} \tag{5}$$

that can well approximate the trajectory of the state vector and then project (3) on that subspace:

$$\mathbf{M}_r\ddot{\boldsymbol{z}} + \mathbf{C}_r\dot{\boldsymbol{z}} + \mathbf{K}_r\boldsymbol{z} = \mathbf{B}_r\boldsymbol{u}(t) \tag{6}$$

where $\mathbf{M}_r = \mathbf{V}^T\mathbf{M}\mathbf{V}$, $\mathbf{C}_r = \mathbf{V}^T\mathbf{C}\mathbf{V}$, $\mathbf{K}_r = \mathbf{V}^T\mathbf{K}\mathbf{V}$, $\mathbf{B}_r = \mathbf{V}^T\mathbf{B}$. In mechanical engineering, the subspace $\mathbf{V}$ is usually chosen from the eigenstates of (3) or by the Guyan method [9].

Moment matching via Krylov subspaces is a new technique [1, 3, 10] that allows to find a low-dimensional subspace with excellent approximating properites for relatively low computational effort.

A straightforward application of Krylov subspace methods to second order ODEs produces a reduced system in the form of a first order system of ordinary differential equations [1, 3, 10], and this is undesirable for structural mechanics. Su and Craig have suggested a modified version of the Arnoldi algorithm that preserves the second order in the reduced model [11]. In both cases, the damping matrix $\mathbf{C}$ takes part in the process of generation of the matrix $\mathbf{V}$.

The main difference of our approach with those in Refs [10, 11] is that the damping matrix is not employed at all during the generation of a low-dimensional basis $\mathbf{V}$, that is, the latter is built as the orthogonolized Krylov subspace $\mathcal{K}\left(\mathbf{K}^{-1}\mathbf{M}, \mathbf{K}^{-1}\boldsymbol{b}\right)$. Nevertheless, the reduced damping matrix has been computed as a projection in (6). Such an approach is based on the assumption that the damping matrix should not play a major role in finding a good subspace $\mathbf{V}$ as the most essential information is contained within the mass and stiffness matrices. In many cases a damping matrix is built up as

a linear combination of mass and stiffness matrices [7], this means that the damping is chosen as mode preserving Rayleigh damping

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}. \tag{7}$$

The unit of α is $1/\mathrm{s}$, the unit of β is $1\,\mathrm{s}$.

The motivation for this choice comes from the fact that by choosing $\mathbf{M}$ and $\mathbf{K}$ such that

$$\mathbf{C} = \mathbf{M}\sum_{b} a_b \left(\mathbf{M}^{-1}\mathbf{K}\right)^{b}, \tag{8}$$

we induce the following properties [7]:

- Damping orthogonality, thus the different modes of the system do not couple through the damping.

- The vibration mode shapes are the same for the damped and undamped system.

- The essential dynamic response is associated with the lowest few modal coordinates and thus suitable for reduction.

Therefore the reduced damping matrix $\mathbf{C}_r = \mathbf{V}^T\mathbf{C}\mathbf{V}$ can be computed directly from the reduced mass and stiffness matrices as

$$\mathbf{C}_r = \alpha\mathbf{M}_r + \beta\mathbf{K}_r, \tag{9}$$

i.e. the parameters α and β are unchanged during the model order reduction process.

1.1 Moment matching for second order systems

After Laplace-transformation of (3) and (4), the transfer function $\boldsymbol{H}(s) = \mathcal{L}(\boldsymbol{y}(s))/\mathcal{L}(\boldsymbol{U}(s))$ can be written as

$$\boldsymbol{H}(s) = \mathbf{L}^T \left(s^2\mathbf{M} + s\mathbf{C} + \mathbf{K}\right)^{-1}\mathbf{B}. \tag{10}$$

For simplicity, let us assume that we only have one output and one input terminal, so that $\boldsymbol{H}(s)$ becomes a scalar, $\mathbf{L}^T$ a vector $\boldsymbol{l}^T$ and $\mathbf{B}$ a vector $\boldsymbol{b}$.

For our method, we also drop the damping term. We expand $H(s)$ by a Taylor series for s^2 at $s_0 = 0$

$$H(s) = \boldsymbol{l}^T \left(s^2 \mathbf{M} + \mathbf{K} \right)^{-1} \boldsymbol{b} \tag{11a}$$

$$= \boldsymbol{l}^T \left(s^2 \mathbf{K}^{-1} \mathbf{M} + \mathbf{I} \right)^{-1} \mathbf{K}^{-1} \boldsymbol{b} \tag{11b}$$

$$= \sum_{i=0}^{\infty} s^{2i} \boldsymbol{l}^T \left(\mathbf{K}^{-1} \mathbf{M} \right)^i \mathbf{K}^{-1} \boldsymbol{b} = \sum_{i=0}^{\infty} m_i s^{2i}. \tag{11c}$$

The m_i are called the moments of the transfer function. We now seek a projection $\mathbf{V}$ that provides a Padé approximation, i.e., that yields the same first q moments for the transfer function of the reduced system.

The Arnoldi algorithm reduces the $n \times n$ matrix $\mathbf{K}^{-1} \mathbf{M}$ to a small $q \times q$ block upper Hessenberg matrix $\mathbf{H}_q$ and during this transformation creates a matrix $\mathbf{V}$ such that

$$\text{colspan}(\mathbf{V}) = \mathcal{K}_m(\mathbf{K}^{-1} \mathbf{M}, \mathbf{K}^{-1} \boldsymbol{b}) \tag{12a}$$

$$\mathbf{V}^T \mathbf{K}^{-1} \mathbf{M} \mathbf{V} = \mathbf{H}_q \tag{12b}$$

$$\mathbf{V}^T \mathbf{V} = \mathbf{I}_q \tag{12c}$$

It can be shown that by using these matrices the corresponding moments in the full and reduced system match up to the mth moment [2].

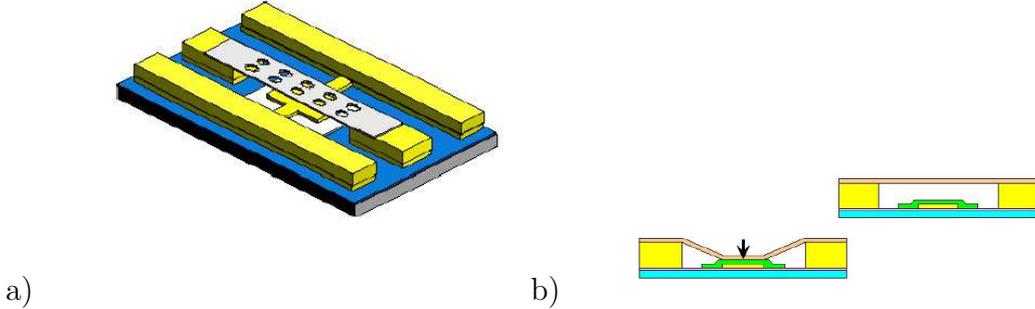a)   b)

Figure 1: a) schematic picture of an RF switch and b) schematic picture of the RF switch actuation. The center bar with holes is electrostatically activated to touch down on the lower electrode.

2   The RF-Switch Model

As an example we used an RF-shunt switch, which is shown in Fig. 1a). The schematic view of the electrostatic activation principle is shown in Fig. 1b). These kinds of switches are used in RF systems and the performance of the switch must be included in the system design path. This can be either done by a FE-Analysis, like it is shown in Fig. 2, where the low frequency modal deflections are reported and the modal functions are given in color encoding. The modal analysis requires approximately 10000 degrees of freedom. This is

Figure 2: The modal analysis for the first three vibrational modes of the switch. The modal analysis was performed with the commercially available FE-code ANSYS. The mode shape functions are color encoded.

even not a very large model and for more complicated structures the number of degrees of freedom might easily reach several 100000. In the following we show the differences in the center deformation of rotation with reduced order models of different number of degrees of freedom compared to the results by solving the full FE model.
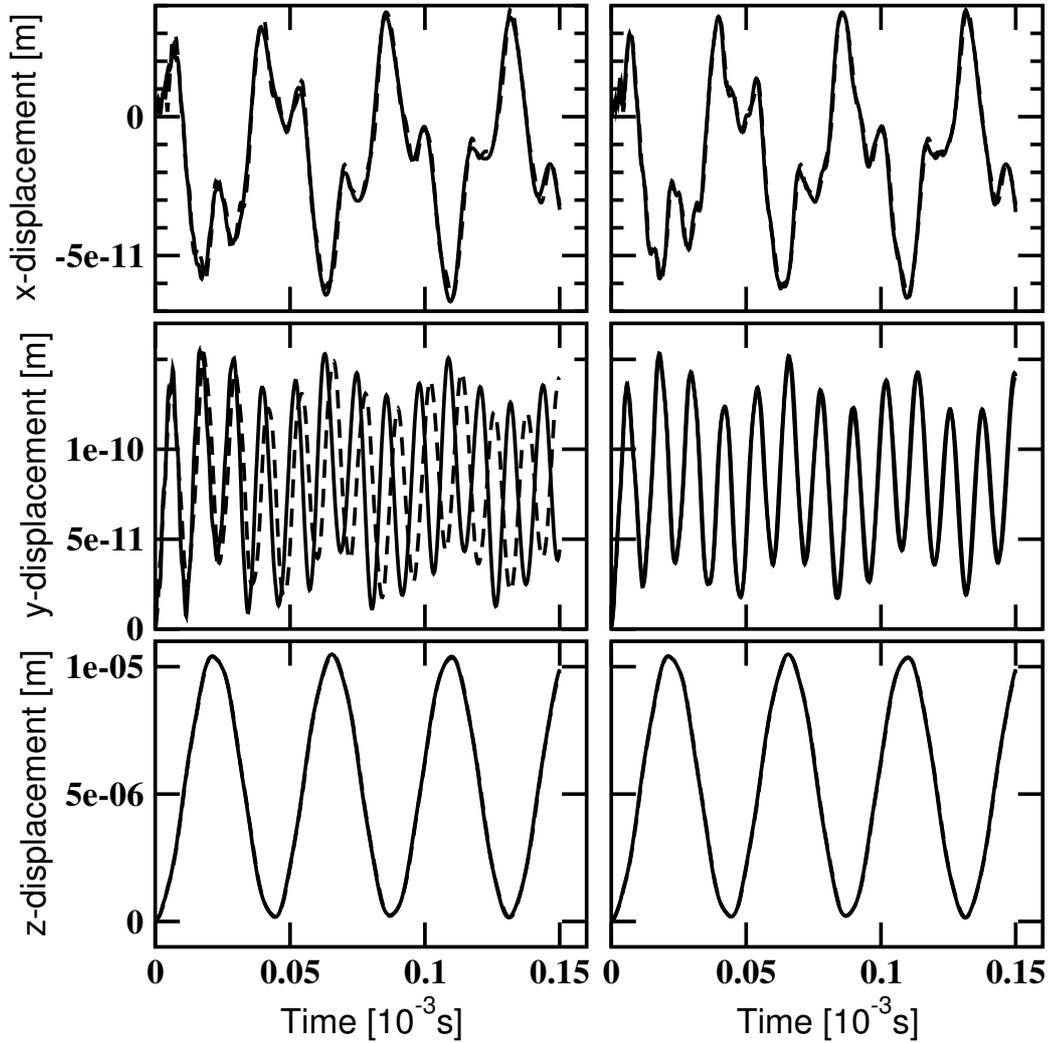
Figure 3: Step response for a force applied to one node at the center of the switch. The displacements in plane (x- and y-direction, upper two viewgraphs) are shown together with the displacement perpendicular to the switch's plane. The left figures show the results for 5 degrees of freedom compared to a FE model while the right figures show the comparison with 30 degrees of freedom. Full lines correspond to reduced order models and dashed lines to FE models.

2.1 Transient simulation

For the transient response a virtual step force was applied to the center of the switch. We compare a reduced order model of 5 degrees of freedom and 30 degrees of freedom with the respective FE solution for the same activation. The result is given in Fig. 3. The in-plane (xy-plane) displacements calculated in the xy-plane with the reduced order model of 5 degrees of freedom do not correspond very well to the FE-model, while the 30 degrees of freedom model shows perfect agreement. The out of plane displacement (z-direction) comparisons instead are quite satisfactory for both numbers of degrees of freedom, and in addition this is the direction of activation of the device. Note that the displacements are given in meters; therefore the small xy-plane displacements are negligible and do not contribute to the performance of the device. In this sense the 5 degrees of freedom model does by far describe the device good enough.

2.2 Harmonic simulation

The harmonic analysis of the FE-model was used to calculate the transfer function of the switch in order to compare it to the computed transfer functions from the reduced order models. Fig. 4 shows the comparison of the center node displacment transfer functions with the FE-model for a reduced order model of 30 degrees of freedom. Both types of models are in good agreement for frequencies up to 1 MHz. Since the operating frequency of the switch is in the range of several tens of kHz, the number of degrees of freedom is far enough for a reduced order model to be used in a system design software. We must stress again the usefullness of the reduced order model in terms of computational time. The computation of a reduced order model takes roughly the same amount of time as the calculation of the full FE-model requires for one timestep.

3 Conclusions

We have presented a novel approach to compute reduced order models of second order damped systems. We showed that this approach works very well for systems where the damping matrix is a linear combination of the stiffness and mass matrix, in addition it greatly outperforms the above mentioned Guyan method.
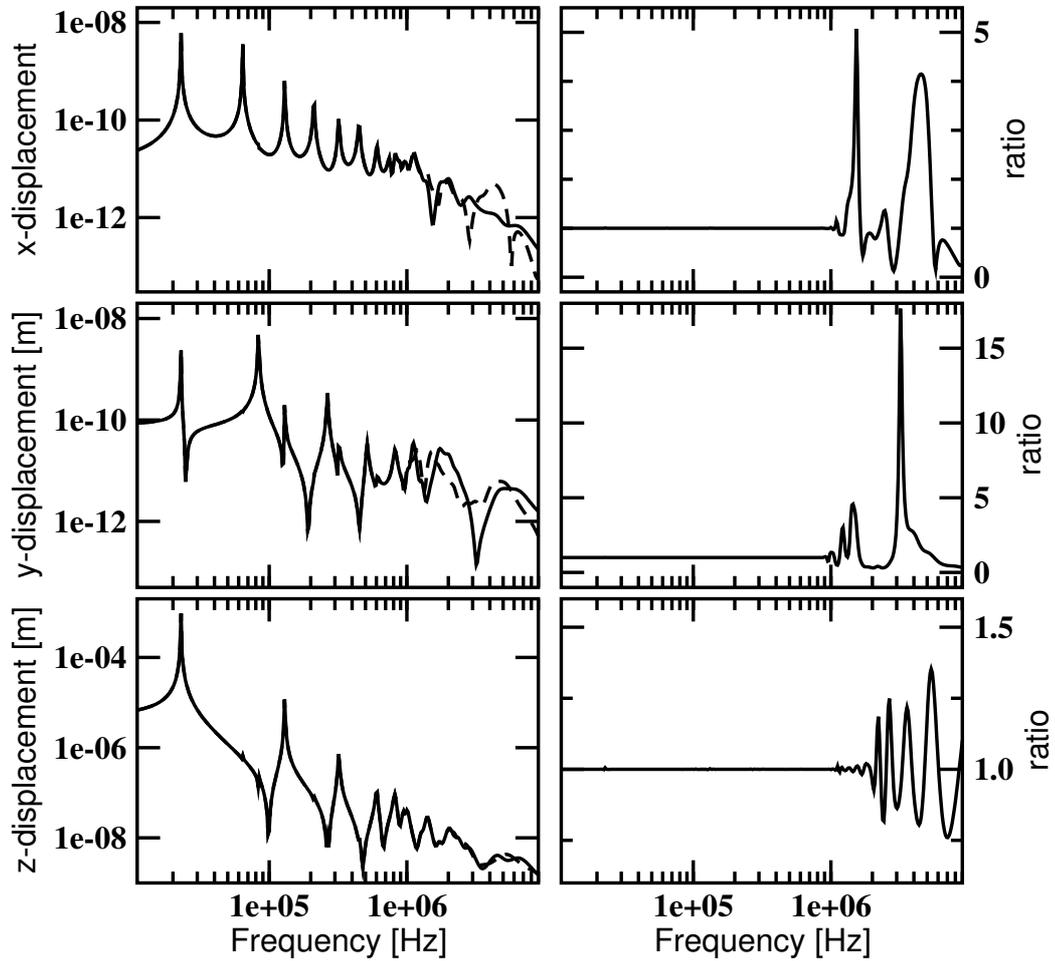
Figure 4: Harmonic response for a force applied to one node at the center of the switch. The transfer functions for the displacements in plane (x- and y-direction, upper two viewgraphs) are shown together with the transfer function for the displacment perpendicular to the switch's plane. The left figures show the 30 degrees of freedom model (full lines) compared to a FE model (dashed lines), while the right figures show ratio of displacements of the reduced order model over the full FEM results.

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4   Acknowledgements

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