Piezoelectric Analysis

- **Fully coupled finite element matrix equation**

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{x}} \\ \ddot{\mathbf{V}} \end{Bmatrix} + \begin{bmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{V}} \end{Bmatrix} + \begin{bmatrix} \mathbf{K} & \mathbf{K}^{Z} \\ \mathbf{K}^{Z^T} & -\mathbf{K}^{d} \end{bmatrix} \begin{Bmatrix} \mathbf{x} \\ \mathbf{V} \end{Bmatrix} = \begin{Bmatrix} \mathbf{F} \\ \mathbf{L} \end{Bmatrix}$$

K: element stiffness matrix

F: force vector

M: element mass matrix

C: element structural damping matrix

$\mathbf{K}^d$: element dielectric permittivity coefficient matrix

L: charge vector

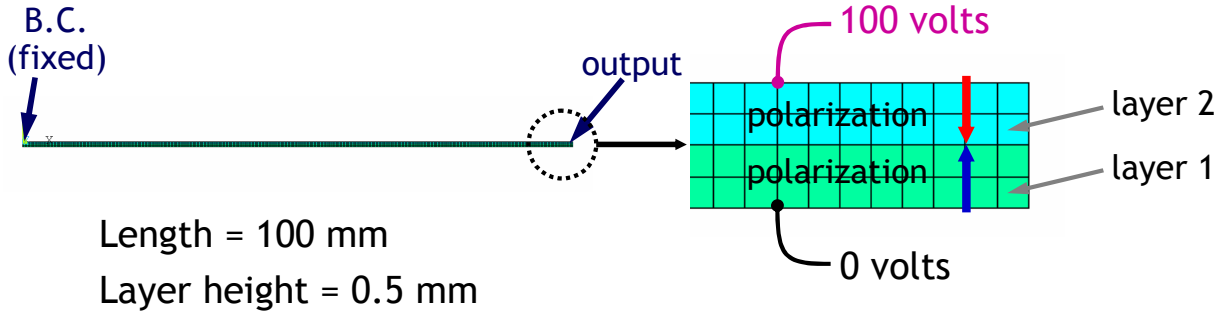
$\mathbf{K}^Z$: piezoelectric coupling matrix

- A fully coupled problem
- Stiffness matrix is indefinite

⇨ TAUCSldlt (LDL^T factorization): symmetric but indefinite

Piezoelectric Example

- **Piezoelectric bimorph actuator**
  : on the application of an electric filed across the beam thickness, one layer contracts while the other expands



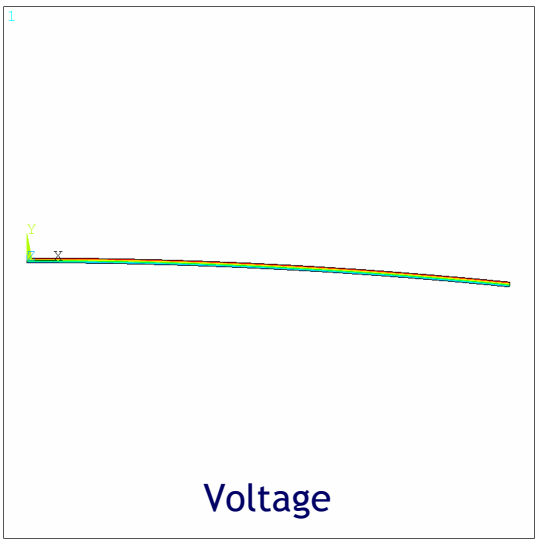
Length = 100 mm

Layer height = 0.5 mm

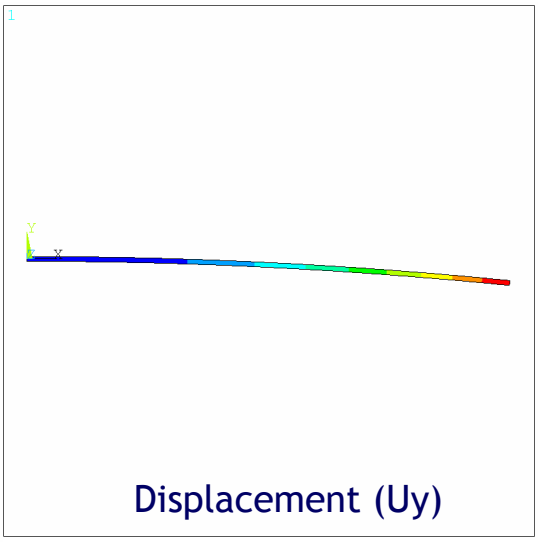
- 1,600 PLANE223 elements
- 5,609 nodes (UX,UY,VOLT)
- 15,207 DOFs

Piezoelectric bimorph actuator

- Static response: actuating mode



Voltage



Displacement (Uy)

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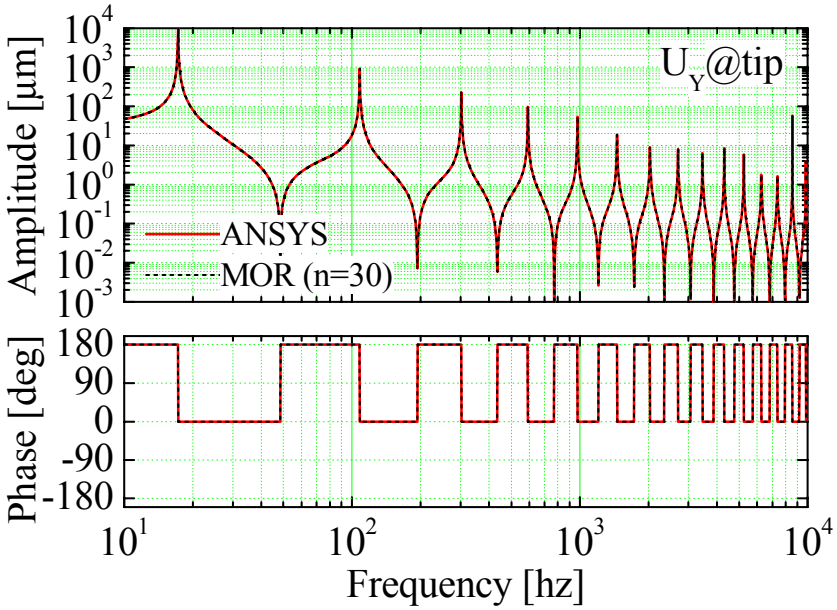
Piezoelectric bimorph actuator

- Frequency response: actuating mode



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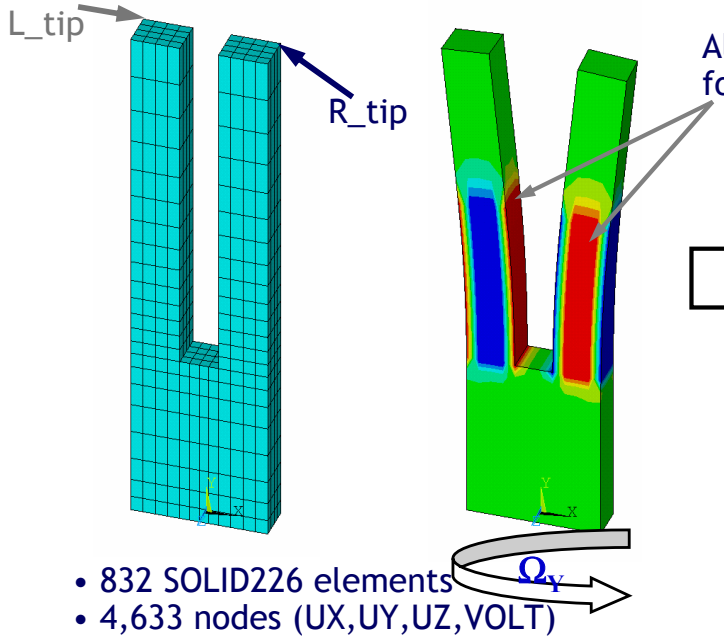
Piezoelectric Example

- **Quartz tuning fork**
  : angular velocity μ-sensor using Coriolis effect



Out-of-plane vibration (U_Z)
proportional to Ω_Y
because of Coriolis effect

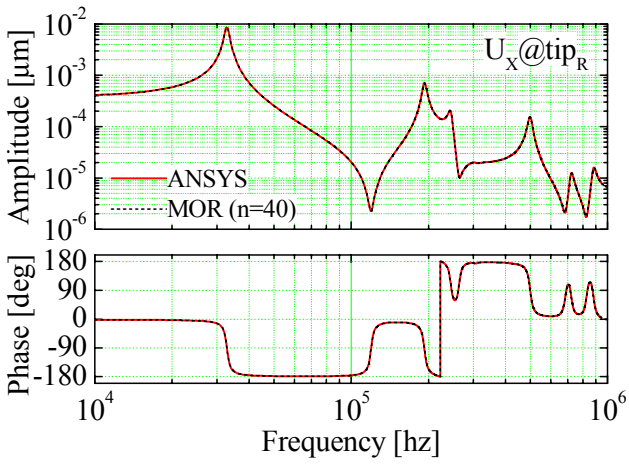
- 832 SOLID226 elements
- 4,633 nodes (UX,UY,UZ,VOLT)

thickness (T) = 350 μm
tine width (W_t) = 450 μm
base width (W) = 1250 μm
tine length (L_t) = 3200 μm
total length (L) = 4800 μm

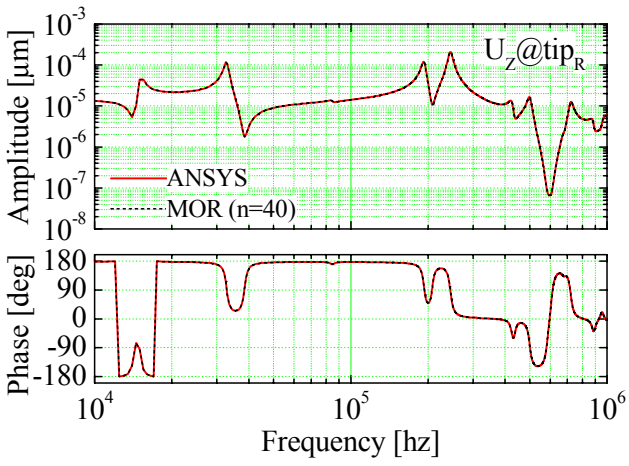
Quartz Tuning Fork

- **Frequency response: in-plane excitation (Ω_Y=0)**

•U_X@tip_R



•U_Z@tip_R

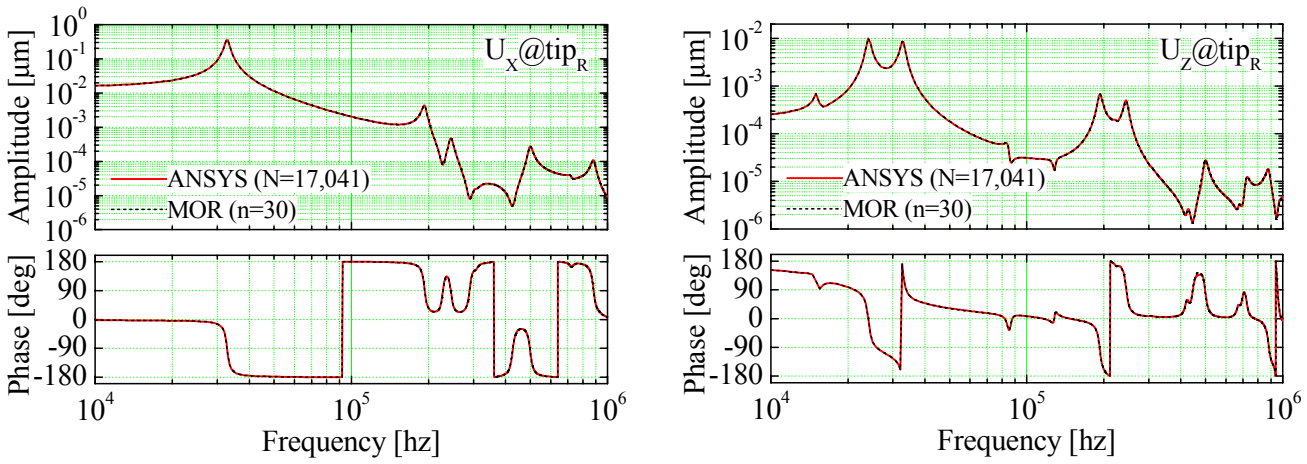


- No. of DOFs: 17,041

Quartz Tuning Fork

- **Frequency response: out-of-plane vibration (Ω_Y=10³ rad/s)**

•U_X@tip_R •U_Z@tip_R



- No. of DOFs: 17,041

Structures with the Coriolis effect

- **Dynamic equation incorporating the effect of rotation**

$$\mathbf{M}\ddot{\mathbf{x}}(t)+(\mathbf{G}+\mathbf{C})\dot{\mathbf{x}}(t)+(\mathbf{K}-\mathbf{K}_C)\mathbf{x}(t)=\mathbf{F}(t)$$

M: global mass matrix

$$\mathbf{M}=\sum_{i}^{Nel}\int_{V_e}\mathbf{N}^T\mathbf{N}\rho dV_e$$

G: global Coriolis matrix

$$\mathbf{G}=\sum_{i}^{Nel}2\int_{V_e}\mathbf{N}^T\boldsymbol{\omega}\mathbf{N}\rho dV_e$$

$\mathbf{K}_c$: global stiffness due to centrifugal force

N: shape function matrix

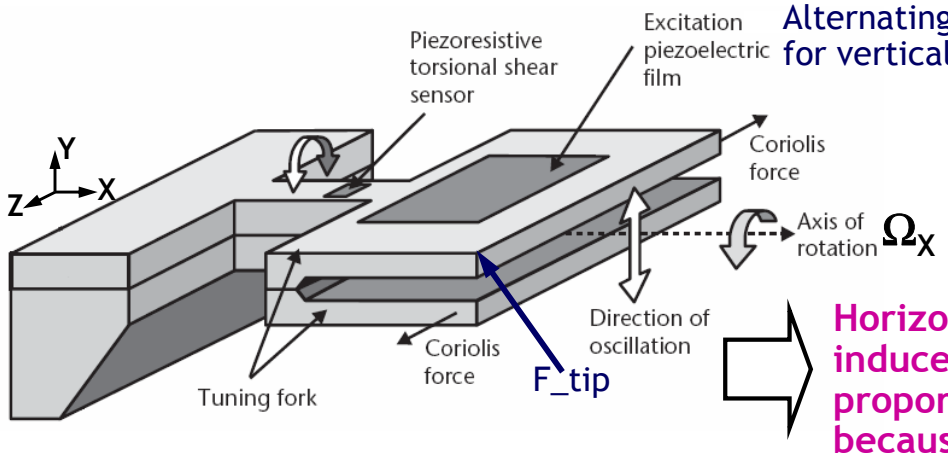
ω: rotational matrix

$$\boldsymbol{\omega}=\begin{bmatrix}0 & -\omega_z & \omega_y\\ \omega_z & 0 & -\omega_x\\ -\omega_y & \omega_x & 0\end{bmatrix}$$

- Because **G** is a constant multiplication of **M** in the case of a structure rotating about an axis, an usual project matrix **V** whose colspan{**V**}=$\mathcal{K}_q(-\mathbf{K}^{-1}\mathbf{M}, -\mathbf{K}^{-1}\mathbf{F})$ is accurate enough to generate reduced systems for analysis with the Coriolis effect!

Piezoelectric Example

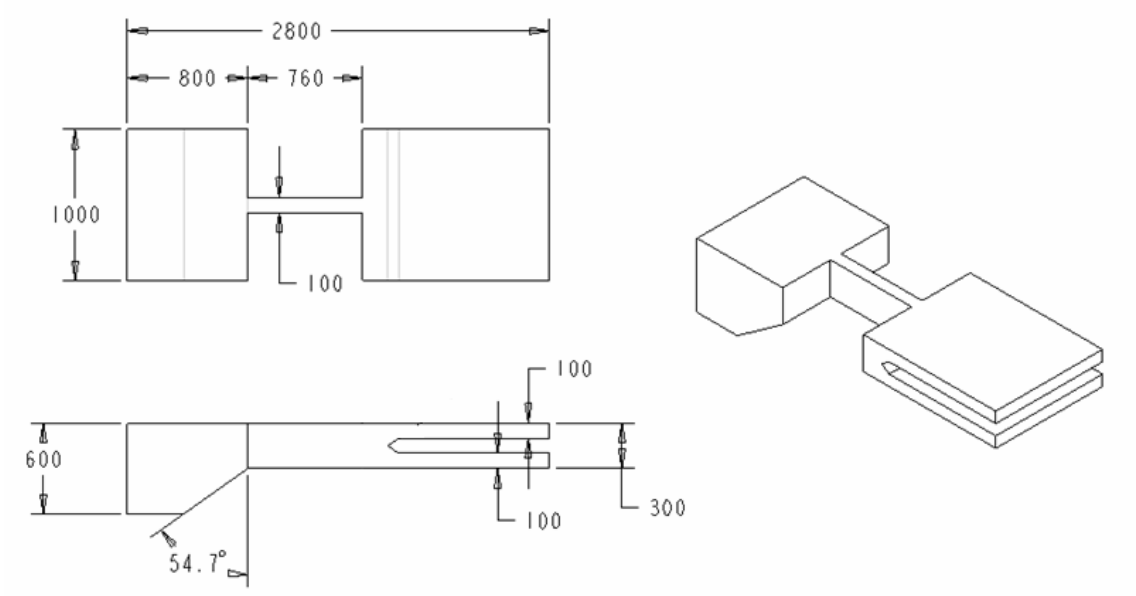
- μ-Gyroscope
  : angular velocity μ-sensor using Coriolis effect



- 1,584 SOLID226 elements
- 8,867 nodes (UX,UY,UZ,VOLT)

μ-Gyroscope

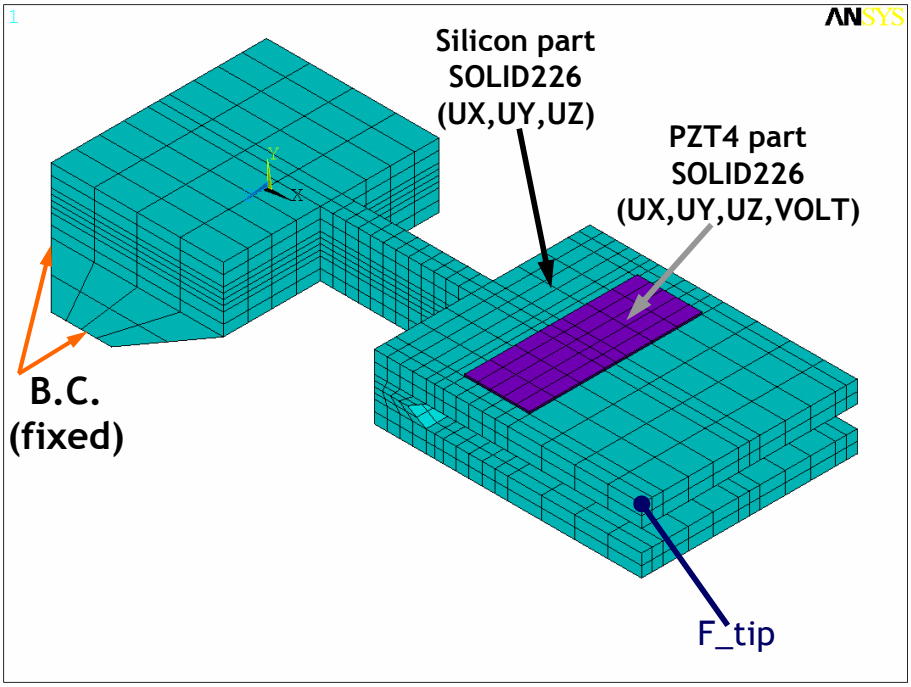
- Dimensions (in μm)



μ-Gyroscope

- Finite element model

- No. of elements: 1,584 (SOLID226)
- No. of nodes: 8,867
- No. of DOFs: 25,449

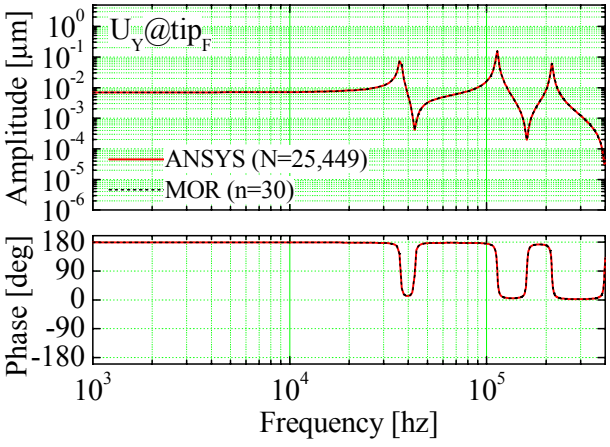


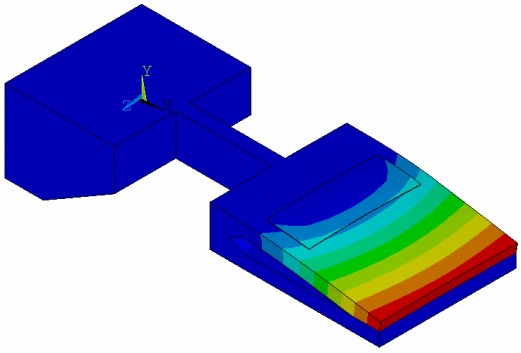
μ-Gyroscope

- Frequency response (Ω_X=0)

  - U_Y@tip_F

vertical excitation





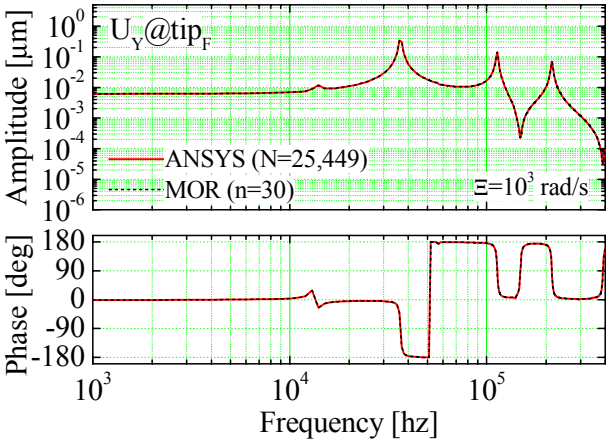
- No. of DOFs: 25,449

μ-Gyroscope

- **Frequency responses (Ω_X=1,000 rad/s)**

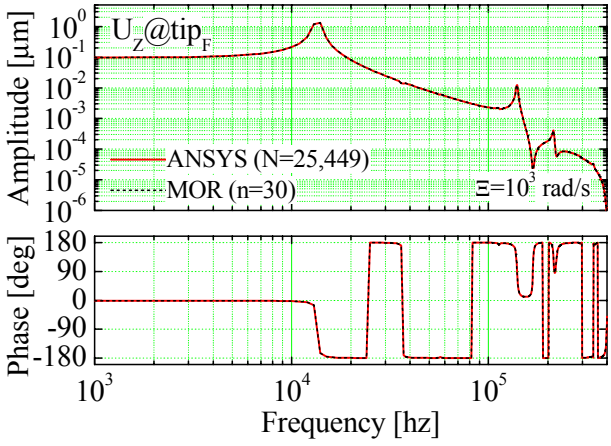
•U_Y@tip_F

vertical excitation



•U_Z@tip_F

Horizontally induced vibration



- No. of DOFs: 25,449

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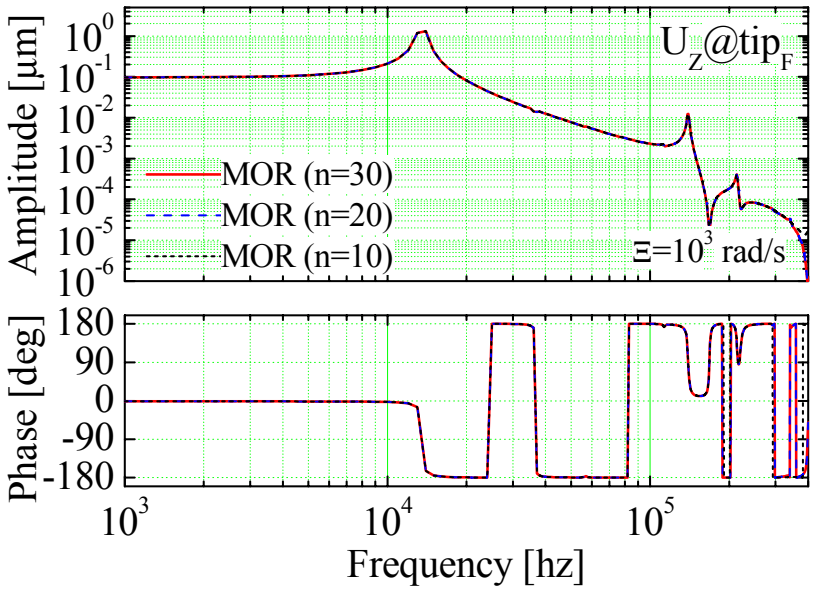
μ-Gyroscope

- **Frequency response: induced vibration (Ω_X=1,000 rad/s)**



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